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Free and Forced Vibrations for High Rise Buildings using Condensation of Matrices

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الملخص:

تعتبر المباني الشاهقة من اهم العلامات الحضارية والثقافية في الدول المتقدمة. لذا كان الاهتمام بتصميمها وتنفيذها المعماري والإنشائي، ونظرا لاختلاف أنظمتها الانشائية لمقاومة الاحمال واختلاف مواد البناء وزيادة قطاعات لمقاومة الاحمال الجانبية، وزيادة عدد الطوابق والمساحات، ادي لزيادة عدد درجات الحرية التي تستغرق وقت كبير لحلها تحليل ديناميكي فزادت المصفوفات وتعقد حلها، لذا كان الحل ايجاد طريقة لتقليل حجم المصفوفة وتقليل عدد درجات الحرية.

الهدف الرئيسي للبحث: تقليل عدد درجات الحرية باستخدام تكثيف المصفوفات، والحل قبل وبعد التكثيف، يتم إجراء مقارنة بين طرق التكثيف الثابتة والديناميكية، أيضا الفرق بين الاختيار الدقيق لدرجات الحرية الرئيسة والمحذوفة ومقارنة النتائج.

لذا، تم التطبيق على نظام إطار صلب ثلاثي الأبعاد، وتم الحل باستخدام FORTRAN ومقارنة النتائج قبل وبعد التكثيف من خلال طرق التكثيف الثابتة والديناميكية. تم إجراء تحليل حر للحصول على التردد الطبيعي الأساسي - الذي يلعب تحليل الاهتزاز الحر دورًا مهمًا في التصميم الإنشائي لأن الشكل الأساسي الوضع هو عنصر مهيمن في الاهتزازات التي تسببها الرياح والزلازل في المباني الشاهقة، حيث يتصرف الهيكل بمرونة.

ثانيًا: كتحليل قسري لاختبار وفهم سلوك الهيكل عند تعرضه لأية أحمال جانبية متوقعة عن الزلازل، فهو يعتبر مصدرًا مهمًا لتصميم آمن واقتصادي. من خلال هذه الحالات الحل قبل وبعد طرق التكثيف سواء الساكنة أو الديناميكية والمقارنة بين طرق التكثيف لمعرفة أدقها، تم التحقق من صحة النتائج باستخدام SAP 2000. تبين أن التكثيف الديناميكي أكثر كفاءة في الترددات وأشكال الأنماط عن التكثيف الساكن، خاصة في المشاكل الديناميكية. يعطي التكثيف الديناميكي للنماذج التي تم تحليلها أقصى انحراف قدره ± 5٪ عن Sap 2000، مع توفير الوقت والجهد في الحل.

ABSTRACT

High buildings are considered one of the most important civilizational and cultural signs in developed countries. Therefore, it was important to pay attention to its architectural and structural design and implementation, and due to the difference in its construction systems to resist loads and the different building materials and the increase in sectors to

resist lateral loads, and the increase in the number of floors, bayes, and spaces, which led to an increase in the number of degrees of freedom that takes a large time to be resolved by dynamic analysis, so the matrices increased and the complexity of their solution, so the solution was Find a way to reduce the size of the matrix and reduce the number of degrees of freedom.
The main objective of the research: is to reduce the number of degrees of freedom using the condensation matrices, and solve before and after condensation. Therefore, the application was made on the 3D steel rigid system, and the solution was done using FORTRAN, and the results before and after condensation were compared by static and dynamic condensation methods. Free analysis was performed in order to obtain the natural fundamental frequency because free vibration analysis plays a vital role in structural design because the basic mode form is a dominant component of vibrations caused by wind and earthquakes in tall buildings, where the structure behaves flexibly.
Second: As a forced analysis to test and understand the behavior of the structure when subjected to any expected lateral loads resulting from earthquakes, it is an important source of information for a safe and economical design. Through these cases, solving before and after condensation methods, whether static or dynamic and comparing between condensation methods to find out the most accurate ones, the results were validated by solving the system using the SAP 2000. The results showed that dynamic condensation is more efficient in frequencies than static condensation, especially in dynamic problems. Dynamic condensation for the analyzed models gives a maximum deviation of ± 5% from Sap 2000

KEYWORDS: Static condensation, Dynamic condensation, Guyan-Iron' reduction, Stiffness matrix, Mass matrix, High-Rise Buildings, Free Vibrations, Forced Vibrations, FORTRAN program.

1. INTRODUCTION

At the beginning of the 20th century, high-rise buildings were designed in general, to achieve an important place in the history of urban architecture and as a rapid response to the increasing population in urban areas. It is almost impossible to imagine a large city without high-rise buildings, most of them have become important symbols in contemporary cities, and have changed the concept of the modern city along with its size and appearance, high-rise buildings are designed today with advanced computer technology and are built with architectural care and structural designs that were not in the past, and the most important factors that enabled the construction of tall buildings are the developments in the following fields: materials, techniques, operating systems (mechanical), construction and analysis systems, but the increase in the height of buildings makes them vulnerable to the lateral loads. The development of building technology played a more important in the development of high-rise buildings, as it was at the end of the 19th century since the discovery of the elevator, the steel to create the (column - beam), that the construction of high-rise buildings began, and high-resistance concrete, foundation, and mechanical systems have made the height of skyscrapers a constant race all over the world. Structural systems were designed in the early 20th century mainly to resist (gravity loads), but today, with the developments in high-resistance materials with the increase in the height of buildings and the decrease in weight, the lateral loads by wind and earthquakes become the main loads, especially in high-rise buildings, there are many structural systems to resist lateral loads.

Due to the variation in its construction systems to resist loads and the difference in construction materials and the increase in sectors to resist lateral loads, and the increase in the number of floors, bayes, and spaces, which led to an increase in the number of degrees of freedom that takes a large time to be resolved by dynamic analysis, so the matrices increased and the complexity of their solution, so the solution was find a way to reduce the size of the matrix and reduce the number of degrees of freedom by the condensation of matrices methods.

In 1964 first one put the basics to reduce the size of the stiffness and mass matrix and also reduce the non-diagonal mass matrix for natural mode analysis is Guyan [1]. Then Irons [2] presented a technique for reducing the elements needed to find eigenvalues for full large matrices on which most of the methods were posteriorly used to condense matrices. The frequency dependent eigenvalue solved by Sturm sequence and some iterations, the accuracy doesn't depend on the choice of masters DOFs except in the rare case when the energy of the system doesn't contribute to the whole set of masters at all, which is referred to as partial vibrations which solved by a new efficient dynamic condensation method without approximation by Leung [3]. when reducing the number of DOFs, it is difficult to apply to complex dynamic structures, so he tried to apply Guyan [1] and remove one degree of freedom at a time, preserving lower frequencies in the reduced eigenvalue problem. The accuracy of the values of natural frequency increases as the frequency ratio fc /f (> 1) increases [4]. Paz. [5] presented a reduction method extension of the static condensation method by an approximate eigenvalue = 0 and applied dynamic condensation to the dynamic matrix, then solving the reduced eigenproblem to determine 1^{st} and 2^{nd} eigenvalues, without requiring matrix inversion or series expansion by reducing secondary DOFs and retaining primary DOFs, by more iterations get exact eigen solutions. The dynamic effects of a substructure's internal DOF can be taken into account in the dynamic analysis of a multi-substructure system, neglecting in some cases, the effects of the internal DOF or higher modes causing major errors [7]. To get an approximate estimate of reduced matrices and makes some modifications to compensate for the inertia effect which is neglected in Guyan, apply a new improved reduced method to improve the accuracy of Guyan[1] by O'Callahan [9]. When selecting accurately the master degrees of freedom can apply the Guyan [1] accurately, the selection of the masters must allow by limits of Guyan [1] to be defined while keeping a minimum of master DOF [10]. Gordis [11] derivated a new technique of the IRS method [8] that the method must respond with a fundamental limitation in the choice of neglected model coordinates, a limitation related to classical reduction methods which try to correct for missing inertia forces. The kept and reduced DOFs in the iterative method by Suarez and Singht are linked with a condensation matrix that is used to get a condensed eigenvalue [12]. While free vibration, the condensed stiffness matrix is exactly the same as the one by Guyan [1], but the two mass matrices are different, and can't accurately preserve the higher modes of interest in the condensed model. But the dynamic condensation method can retain both the lower and higher modes of interest in the resulting model with high accuracy but must choose the frequencies to produce the condensation and to perform the result in the dynamic responses of a structure it must choose frequencies closer to the actual applied loads [13]. 1st compute the damping matrix alone, but the mass and stiffness matrices get from the normal frequency response functions by using the least-squares method [14]. The IRS method is extended by the equivalent transformation based on dynamic condensation rather than static condensation as an iterative method, provides a reduced model which reproduces a subset of the model of the full matrices [15]. Naguib. [16] created the FORTRAN program to solve dynamic problems by the three methods of condensation (Static, Dynamic, and Modified), applied in dynamic structures as free vibration, it was emphasized that it is the nearest values of real values that result from dynamic condensation. An iterative method by Kim [17] to solve the eigenproblem for large structures has combined dynamic condensation, modified subspace iteration, and modal reduction, after the selection of master degrees of freedom The master advantage that several eigenpairs can be with solution accuracy through a combined reduction, it has also made modifications to the dynamic condensation method to solve dynamic analysis, incorrectly selecting master degrees of freedom has little effect on the overall matrices. It was possible to apply the dynamic condensation method to get the natural frequencies and reduce the number of mode shapes. The measured frequencies can be used as the initial assumption in the dynamic condensation to find an exact solution for the real eigenvalue and find the transformation matrix between the experimentation, and not experimentation [18]. An iterative technique with three advantages: (1) The convergence is much faster than all past methods, especially when the eigenvalues of the reduced model are close to the full model. (2) The convergence proved simple. (3) it is unnecessary to determine the stiffness and mass matrices also the eigen solutions of the reduced model in every iteration [19]. The lower modes converge very quickly than the higher modes and the master co-ordinates should be chosen to give an accurate static reduction [21]. The condensed models by dynamic condensation can keep the selected number of lower natural frequencies and corresponding modes accurately as for the known loads, the intense model from the dynamic condensed method can show the dynamic responses of the structure with very good accuracy, but the unknown loads but within the frequency range, the models produced using the optimum frequencies of the approximately applied loads have better than the intense models by Guyan [22]. When the dynamic condensation is independent of the eigenvalues of the reduction, it's not significant to determine them in every iteration, this makes the iterative very active mostly when having a large number of kept DOFs, so the dynamic condensation matrix is an advantage of the system and is not influenced by external forces [23]. An iterative method that essentially uses orthonormalized complex eigenvectors of the unsymmetrical system, the eigen-solution of the reduced-order model with master DOFs is obtained by the Lanczos algorithm. The model reduction procedure is further used in substructure and eigenvalue analysis of large-size unsymmetric systems [27]. Dynamic condensation used an algorithm to fast estimate some low eigenvalues and corresponding eigenvectors of the large structures by reducing the original structural model to a smaller one [28]. One of the condensation methods is based on

the generalized inverse of the matrix while ignoring the damping and inertia forces on all DOFs of the full model, these algorithms are considered static condensation, one advantage is that specified forms of reduced stiffness, mass, and damping matrices can be directly obtained from the reduced model, these reduced matrices are very useful in dynamic analyses, comparing approximations from assumptions, condensation matrices, and reduced matrices, with the generalized inverse of the matrix, the method defined in the displacement space is extended and one variant is derived [29]. An iterative method for the dynamic condensation of non-classically damped systems based on the dynamic equations defined in displacement and state spaces, neglecting the effect of the damping on the dynamic condensation matrix, the result from the dynamic condensation matrix is corresponding to that of the undamped model, it is easy in the dynamic analysis, and the iteration of this method will not converge to the exact values, especially for the damping parameters, to solve this problem [30]. Xia, and Lin [31] proposed an improvement on this dynamic condensation method by iterating improved reduced system IIRS to modify the iterative transformation matrix and achieve faster convergence. It is possible to use the dynamic condensation method for the reduction in active vibration control of large and flexible structures, dynamic reduction has many advantages, especially its accuracy in the high-frequency range [34]. Dynamic condensation to extend analysis at higher frequencies by selecting master DOFS and condensing the structure matrices on those DOFS [35]. An iterated improved reduced system IIRS based on Friswell's for un-damped and non-classically damped structures, and it is an effective method because the highly accurate Eigen-properties from the repeatedly condensed matrices can be obtained without expensive computational cost [36]. For the problem of inverting a matrix, a closed-form solution is applied, the structures include sub-models each having a different repeated pattern, the easily inverse using regular matrices requiring a smaller amount of computational time, by dynamic condensation and the matrix inversion, the eigen-solution is performed on matrices of lower dimensions [38]. Kim and Kang [39] developed a multi-level condensation method to improve the efficiency of traditional matrix condensation by proposing an efficient analytical model for a super-tall mega-frame building condensed analytical model with minimal DOFs to efficiently predict the global structural behavior of mega frame structures, by a simple model which presents global structural behavior, rather than a complicated finite element model. A new damage detection method that uses only one mode shape and its corresponding eigenvalue of a structure to conduct damage detection, updated the condensed stiffness matrix through an iterative damage detection procedure. by the transformation matrix of the dynamic condensation technique in a coupled recursive procedure to update damage indices. Also, updated stiffness matrix, mode shapes, and frequencies of all structures [42]. There are various methods to reduce matrix, the original one is involving the well-known Static Condensation method, requiring multiplication and inverse of several sub-matrices by the simple Cholesky analysis to get the reduced stiffness matrix [43]. To solve FE models with more than millions of DOFs, it would be valuable to develop a more efficient algebraic dynamic condensation method employing multi-level sub structuring [44]. A non-iterative method, which is known as Maclaurin Expansion of the frequency response function in Laplace Domain' (MELD) applied for dynamic reduction of non-classically damped structures as a non-iterative method is an efficient method to reduce the size of the matrices by removing the unimportant DOFs because the (DOFs) becomes larger and makes the analyzing complexity [46]. The dynamic condensation method transformed the global vibration equation into a reduced one with a much smaller size, from the reduced vibration equation with numerical time integration methods get the structural responses efficiently, derived directly from the response sensitivities from the reduced vibration equation without extra master DOFs [48].

The main object of this paper is to examine the effect of Static and Dynamic condensation on the dynamic analysis of high-rise buildings . 3-D rigid frames with many DOFs are analyzed by using condensation techniques based on the Guyan-Irons reduction. The results are compared before and after condensation, whether (static or dynamic) on high-rise buildings using the FORTRAN program, and the results were verified by SAP 2000.

2. METHODOLOGY

The dynamic equations of equilibrium are written as a set of linear second-order differential equations:

$$[M]\{\ddot{X}(t)\} + [C]\{\dot{X}(t)\} + [K]\{X(t)\} = \{F(t)\} \quad (1)$$

The reduced dynamic eqn. of equilibrium is written as:

$$[M_R]\{\ddot{Z}(t)\} + [C_R]\{\dot{Z}(t)\} + [K_R]\{Z(t)\} = \{F_R(t)\} \quad (2)$$

Where: M_R, C_R *and* K_R are the mass, damping, and stiffness matrices, of the reduced order model respectively, and F_R is the force vector of the reduced model. They are defined as:

$$[M_R] = [T]^T [M][T] \quad \& \quad [C_R] = [T]^T [C][T]$$

$$[K_R] = [T]^T [K][T] \quad \& \quad [F_R] = [T]^T [F][T]$$

Dynamic properties are preserved in the reduced model so it is useful in dynamic analyses, especially when iterating.

I. Static Condensation Method

A- Static Condensation Method on the Static Problem

In static structural analysis it become necessary to reduce the stiffness matrix $[K]$ only. By Guyan [1] and Irons [2] for the removed unwanted DOFs by separating the number of DOFs into secondary and primary. The stiffness Equation for structure:

$$\begin{bmatrix} [Kss] & \cdots & [Ksp] \\ \vdots & \ddots & \vdots \\ [Kps] & \cdots & [Kpp] \end{bmatrix} \begin{bmatrix} \{u_0\} \\ \vdots \\ \{u_p\} \end{bmatrix} = \begin{bmatrix} \{0\} \\ \vdots \\ \{F_p\} \end{bmatrix} \quad (3)$$

Let: $$\begin{bmatrix} [Kss] & [Ksp] \\ [Kps] & [Kpp] \end{bmatrix} \begin{bmatrix} \{u_s\} \\ \{u_p\} \end{bmatrix} = \begin{bmatrix} \{0\} \\ \{F_p\} \end{bmatrix} \quad (4)$$

Where $\{u_s\}$ is the displacement vector corresponding to the DOFs to be reduced and $\{u_p\}$ is the vector corresponding to remaining p independent DOFs. By assume external forces= zero at secondary DOFs. By multiplication of matrices in Eqn. (1) expands into Eqns. (5) & (6).

$$[Kss]\,\{u_s\} + [Ksp]\,\{u_p\} = \{0\} \quad (5)$$

$$[Kps]\,\{u_s\} + [Kpp]\,\{u_p\} = \{F_p\} \quad (6)$$

$$\{us\} = [\,\bar{T}\,]\,\{u_p\} \quad (7)$$

$$[\,\bar{T}\,] = -[Kss]^{-1}[Ksp] \quad (8) \qquad [\,\bar{K}\,]\,\{u_p\} = \{F_p\} \quad (9)$$

Where, $[\,\bar{T}\,]$ is the transformation matrix, and, $[\,\bar{K}\,]$ is the reduced stiffness matrix,

$$[\,\bar{K}\,] = [Kpp] - [Kps]\;[Kss]^{-1}\,[Kps] \quad (10)$$

When, $\{u\} = [\,T\,]\,\{u_p\}$ (11)

$$\{u\} = \begin{bmatrix} \{u_s\} \\ \{u_p\} \end{bmatrix} \quad \& \quad [\,T\,] = \begin{bmatrix} [\,T\,] \\ [\,\mathrm{I}\,] \end{bmatrix} \quad (12)$$

Sub. in Eqns. (11) and (3) into Eqn. (4) and the transpose of [T] results in:

$$[T]^T\,[K\,]\,[T\,]\{u_p\} = [\,[T]^T\;[\,\mathrm{I}\,]\,]\begin{bmatrix} \{0\} \\ \{F_p\} \end{bmatrix}$$

$$[T]^T\,[K\,]\,[T\,]\{u_p\} = \{F_p\}$$

$$[\,\bar{K}\,] = [T]^T\,[K\,]\,[T\,] \quad (13)$$

By solving, (1), (12) into (4) the stiffness equation has been reduced to:

$$\begin{bmatrix} [\,I\,] & -[\ \bar{T}\ ] \\ [0] & [\ \bar{K}\ ] \end{bmatrix} \begin{bmatrix} \{u_s\} \\ \{u_p\} \end{bmatrix} = \begin{bmatrix} \{0\} \\ \{F_p\} \end{bmatrix} \quad (14)$$

Guyan ignored dynamic effect, so this method is exact for static problems only. However, widely used in many static and dynamic problems. So, Guyan is considered initial approximation of exact dynamic condensation. Error in Guyan depends upon ratio of cut frequency to interested frequency. The higher ratio, the more accurate the reduced model.

B- **Static Condensation Method on the Dynamic Problem**

To reduce the mass$[M]$and damping $[C]$matrices, it is assumed that the same static relationship between secondary and primary DOFs remains effective in the dynamic problem.

The reduced mass and damping $[C]$matrices are given by:

$$[\ \bar{M}\ ] = [T]^T\ [M]\ [T\ ] \quad (15)$$

Where, $[\ \bar{\mathrm{M}}\ ]$ is the reduced mass matrix.

$$[\ \bar{C}\ ] = [T]^T\ [C\ ]\ [T\ ] \quad (16)$$

Where, $[\ \bar{C}\ ]$ is the reduced damped matrix.

The reduction of the mass and damping matrices can be justified by potential elastic energy V and kinetic energy KE of the structure, as:

$$V = \frac{1}{2}\ \{\,u\,\}^T[\mathrm{K}]\ \{\mathrm{u}\} \quad (17)$$

$$KE = \frac{1}{2}\ \{\,\dot{u}\,\}^T[\mathrm{M}]\ \{\dot{u}\} \quad (18)$$

Virtual work δW_d by damping forces F_d=[C]$\{\dot{u}\}$ corresponding to virtual displacement $\{\ \delta_{\mathrm{u}}\}$ as:

$$\delta W_d = \{\ d_u\ \}^T[\mathrm{C}]\ \{\dot{u}\} \quad (19)$$

By the transformation Eq. (11) the results are

$$V = \frac{1}{2}\ \{\,\mathrm{u_p}\,\}^{\mathrm{T}}\ [\mathrm{T}]^{\mathrm{T}}\ [\mathrm{K}\ ]\ [\mathrm{T}\ ]\{\mathrm{u_p}\} \quad (20)$$

$$KE = \frac{1}{2}\ \{\ \dot{\mathrm{u}}_{\mathrm{p}}\}^{\mathrm{T}}\ [\mathrm{T}]^{\mathrm{T}}\ [\mathrm{M}\ ]\ [\mathrm{T}\ ]\{\ \dot{\mathrm{u}}_{\mathrm{p}}\} \quad (21)$$

$$\delta W_d = \{\delta \mathrm{u_p}\ \}^T[\mathrm{T}]^{\mathrm{T}}\ [\mathrm{C}\ ]\ [\mathrm{T}\ ]\{\ \dot{\mathrm{u}}_{\mathrm{p}}\} \quad (22)$$

Substitution of $[\ \bar{K}\ ]$, $[\ \bar{M}\ ]$, and $[\ \bar{C}\ ]$ from Eqns. (13), (15), and (16) get:

$$V = \frac{1}{2}\ \{\,\mathrm{u_p}\,\}^{\mathrm{T}}\ [\ \bar{K}\ ]\ [\mathrm{T}\ ]\{\mathrm{u_p}\} \quad (23)$$

$$KE = \frac{1}{2}\ \{\ \dot{\mathrm{u}}_{\mathrm{p}}\}^{\mathrm{T}}\ [\ \bar{M}\ ]\ [\mathrm{T}\ ]\{\ \dot{\mathrm{u}}_{\mathrm{p}}\} \quad (24)$$

$$\delta W_d = \{\delta \mathrm{u_p}\ \}^T[\ \bar{C}\ ]\ [\mathrm{T}\ ]\{\ \dot{\mathrm{u}}_{\mathrm{p}}\} \quad (25)$$

Static condensation drives errors when applied to dynamic problems.

II. Dynamic Condensation Method

In dynamic condensation methods take into account the effects of inertia of ignored DOFs. Because the inertia is related to the inverse of the dynamic stiffness matrix, they cannot be obtained directly. It an extension of the static condensation method. By assuming the first eigenvalue ω_1^2 is approximate value or set it equal= 0.
The dynamic matrix: $[D_1] = [K] - \omega_1^2\ [M\ ]$, to get ω_1^2 and ω_2^2. The matrix inversion or series expansion does not require. The equations of free motion form:

$$\begin{bmatrix}[Mss] & [Msp]\\ [Mps] & [Mpp]\end{bmatrix}\begin{bmatrix}\{\ddot{u}_s\}\\ \{\ddot{u}_p\}\end{bmatrix}+\begin{bmatrix}[Kss] & [Ksp]\\ [Kps] & [Kpp]\end{bmatrix}\begin{bmatrix}\{u_s\}\\ \{u_p\}\end{bmatrix}=\begin{bmatrix}\{0\}\\ \{0\}\end{bmatrix} \tag{26}$$

Let $\{u\} = \{U\}\sin\omega t$

So, $$\begin{bmatrix}[Kss]-\omega_1^2[Mss] & [Ksp]-\omega_1^2[Msp]\\ [Kps]-\omega_1^2[Mps] & [Kpp]-\omega_1^2[Mpp]\end{bmatrix}\begin{bmatrix}\{u_s\}\\ \{u_p\}\end{bmatrix}=\begin{bmatrix}\{0\}\\ \{0\}\end{bmatrix} \tag{27}$$

From Guess Jordan: $$\begin{bmatrix}[I] & -[\ \bar{T}_i\ ]\\ [0] & [D_i]\end{bmatrix}\begin{bmatrix}\{U_s\}\\ \{U_p\}\end{bmatrix}=\begin{bmatrix}\{0\}\\ \{0\}\end{bmatrix} \tag{28}$$

$\{U_s\} = [\ \bar{T}_i\ ]\ \{U_p\}$ (29). $\{\ U\ \}_i = [\mathrm{T}]\ \{U_p\}$ (30)

$$[\ T_i\ ] = \begin{bmatrix}[\ \bar{T}\ ]\\ [\ \bar{I}\ ]\end{bmatrix}\quad \& \quad \{U_i\} = \begin{bmatrix}\{U_s\}\\ \{U_p\}\end{bmatrix} \tag{31}$$

To reduce the Mass matrix: $[\ \bar{M}_i\ ] = [\ T_i]^T\ [M]\ [T_i]$ (32)

And to get stiffness matrix: $[\ \bar{K}_i\ ] = [\ \bar{D}_i\ ] + \omega_i^2\ [\ \bar{M}_i\ ]$ (33)

The reduced Eigen problem from: $[\ \bar{K}_i\ ] - \omega_i^2\ [\ \bar{M}_i\ ]\ |\ \{u_p\} = \{0\}$ (34)

3. Selection of Master the Degrees of Freedom

The total DOFs of a full model are divided into masters and slaves in the dynamic condensation. There are conditions for selecting the masters because any error in selecting leads to a big error while condensing matrices and the results will be inaccurate. The selection must make the reduction model as accurate as possible.

- Selection. of the Masters of The Guyan Condensation

The eigenvalue range is (0, ω_c^2) while ω_c^2 is the lowest eigenvalue of the slave model. The approximate error of the eigenvalues from Guyan is inversely proportional to the eigenvalue ω_c^2. So, the bigger eigenvalue ω_c^2 the more accurate Guyan.
-The degrees at maximum relative displacement occurs should be selected as masters.
- At centralized mass, DOFs should be selected as master DOFs.

Physical-Type Condensation

1-Levy (1971): Select the DOFs that have;
(a) largest entries in the mass matrix
(b) largest movements in the modes of interest.
2- Ramsden and Stocker (1969): Selected the masters DOF that with larger mass and flexible reasonably relative to other mass concentrations and fixed constraints.
3- Downs (1980): The masters DOFs should be displacements rather than rotations.

4. Case Study by FORTRAN Program:

Example 1: Verification Example

For the shown space frame (three-dimensional frame) in Fig.1:

TABLE 1. Input data for space frame

Quantity	Members 1,3	Members 2,4
Modulus of elasticity (psi)	$30\ x10^6$	$30\ x10^6$
Modulus of rigidity (psi)	$12\ x10^6$	$12\ x10^6$
Distributed mass (Ib. sec^2/in^2)	0.2	0.1
Cross-sectional y moment of inertia (in^4)	200	64
Cross-sectional z moment of inertia (in^4)	200	64
Torsional constant (in^4)	40.0	12.8
Cross-sectional area (in^2)	50	28
Cross-sectional moment of inertia (in^4)	205	68

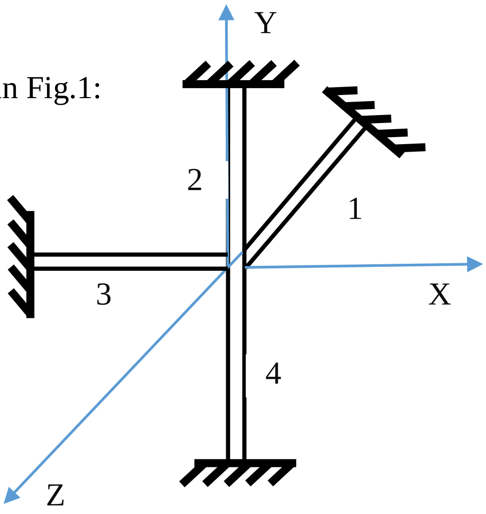

Fig. 1 Space Frame Example [36].

TABLE 2 Eigen-Values for space frame.

	Mario Paz	FORTRAN program	Error%
ω_1^2	6420	6507.062	1.34%
ω_2^2	6520	6533.963	0. 2137%
ω_3^2	7833	7871.219	0.488%
ω_4^2	174600	174568.500	0.018%
ω_5^2	239600	239744.800	0.060%
ω_6^2	267500	267846.900	0.13%

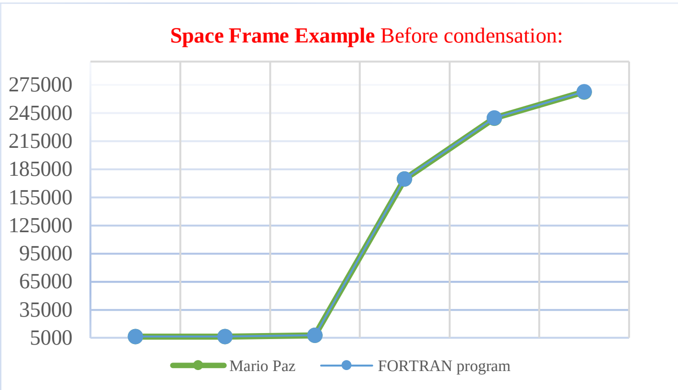

Fig. 2Comparison between FORTRAN and M. Paz for Space Frame Example before condensation.

After Condensation of Coordinates $\mathbf{d_1}$, $\mathbf{d_2}$ & $\mathbf{d_3}$

TABLE 3 Eigen-Values. Before Cond., After Static and Dynamic Condensations for Space

	Before Cond.	Static Cond.	Error %	Dynamic Cond.	Error %
ω_1^2	6507.062	6533.963	0.4117	6507.062	0.0
ω_2^2	6533.963	6566.035	0.4885	6533.963	0.0
ω_3^2	7871.219	7968.901	1.2258	7871.219	0.0
ω_4^2	174568.5				
ω_5^2	239744.8				
ω_6^2	267846.9				

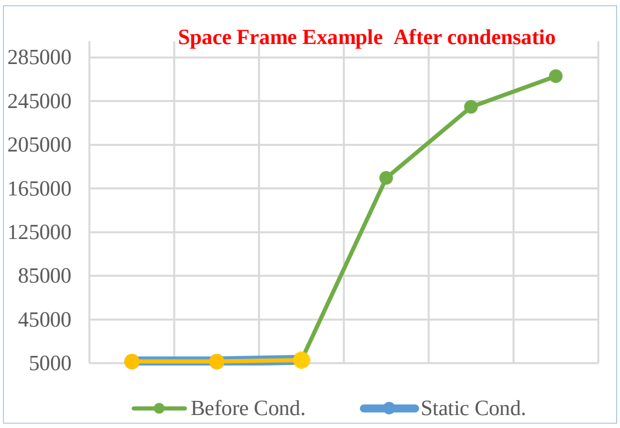

Fig. 3 Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for Space Frame Example.

Example 2

For a 20-story shear building with a height of 4 m for the story floor and 3 m for the repeated stories as shown in the Fig. (4) according to the moments of inertia of reinforced concrete column =.00032552 m^4 with the uniformly mass weight for each story= .6116208 t. sec^2/m. Calculated by the FORTRAN program the natural frequency results before and after the static and dynamic condensation. Then the results were compared, as shown Fig. (5), between the natural frequencies before and after the static and the dynamic condensation by eliminating 50% & 25% of DOFs.

From the in Fig. (5), we find that the greater the number of degrees of freedom removed, the accuracy of the results decreases, that is, when removing 25% is more accurate and closer to the truth than removing 50% of the degrees of freedom. The dynamic condensation is closer to the correct and more accurate than the static condensation, especially since it is a dynamic analysis.

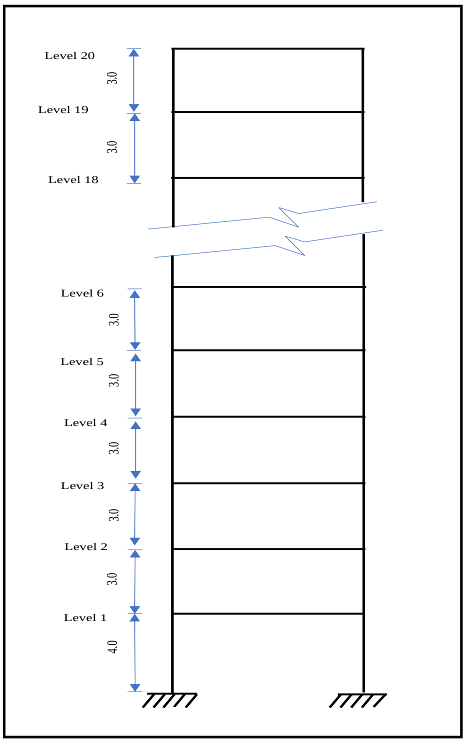

Fig. 420-story shear building

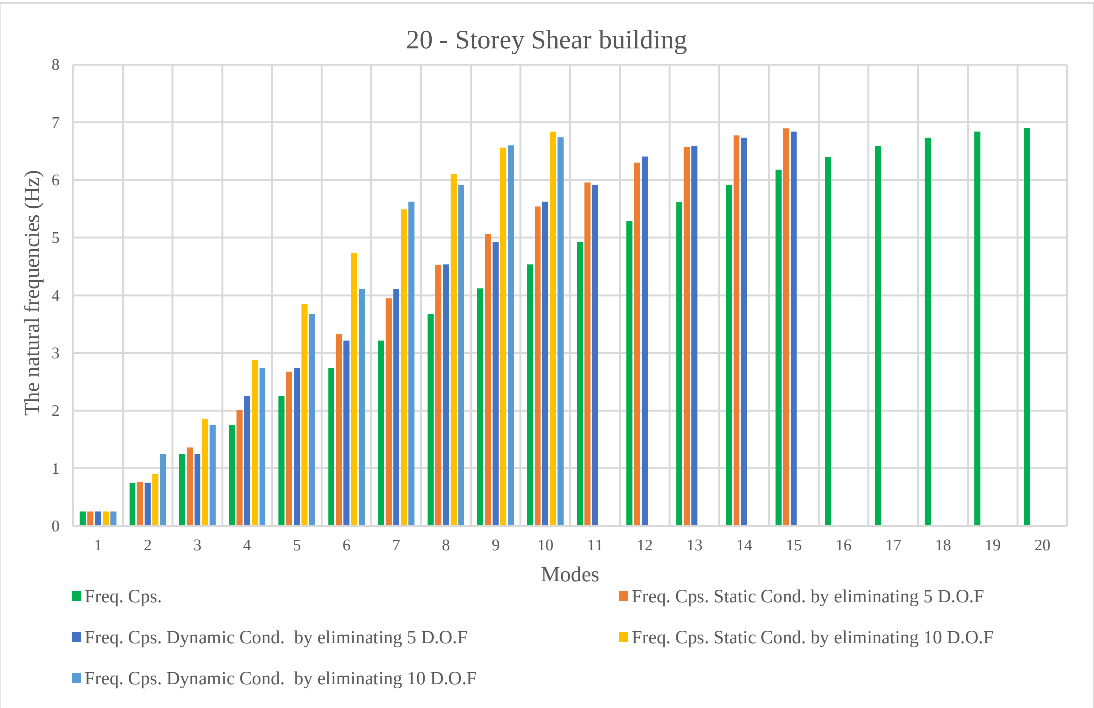

Fig.5 The natural frequencies (Hz) of a 20-storey shear building before and after static and dynamic condensation after reduce 5 & 10 DOFs.

Example 3

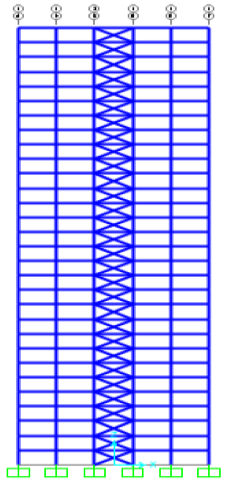

Fig. 6 2D steel rigid frame.

For the application of static and dynamic condensation on high-rise buildings, a Two-dimensional steel rigid frame system to resist gravity load as the main part and the lateral load as a secondary part was chosen, and it was analyzed free analysis, and the results were compared between static and dynamic condensation for free analysis and forced analysis**.** The results were validated before condensation with the SAP 2000 program.

The case study dependent on a 30-story high-rise steel frame building with reinforced concrete slabs and brick walls used as a model as shown in Fig. (6). The rigid frames are arranged in X and Z directions consisted of 30 stories and the height of the story was H= 3 m and arranged in the X-direction (5x6m). The steel frames with a grade of steel S355 were used with Fy = 355MPa and Fu=480Mpa, used the beam section with sectors IPE (500), and divided the steel columns into 2 groups, the 15 below stories were with sections H (400x551) and the next 15 stories with H (400x467). The steel bracing used to resist the lateral loads as a pipe of the steel with a radius R= 15.0cm and thickness t = 5mm. The solid slab is reinforced concrete with standard concrete grade C30/37 and it is a square slab with a cover weight = 0.15t/m^2. The mass of the building is from the D.L and 25% from the L.L as (Model Mode) in SAP2000. A FORTRAN program for static and dynamic condensation is created to generate the required data file for SAP2000. The natural frequency (cps) results were calculated by the FORTRAN program before and after the static and the dynamic condensation at remain 90, 80, 70, 60, 50, and 40% of DOF. respectively. Then compared 1st 10-frequencies as shown in the Tables. (4), (5), (6), (7), (8) & (9) between the natural frequencies before and after static and dynamic condensation.

TABLE 4. Comparison between FORTRAN Sol. Before, After Static and Dynamic Condensations by remaining 90**%** D.O.F

MODE	Frequencies (cps)				
	Exact Sol.by FORTRAN	Static condensation by remaining%90 D.O. F	Error %	Dynamic condensation by remaining %90D.O.F	Error %
1	**0.0355**	0.0356	0.2809	0.0355	0.000
2	**0.0528**	0.0531	0.5649	0.0528	0.000
3	**0.2087**	0.2092	0.2390	0.2087	0.000
4	**0.2375**	0.2385	0.4192	0.2375	0.000
5	**0.3867**	0.3877	0.2579	0.3867	0.000
6	**0.4681**	0.4731	1.0568	0.4681	0.000
7	**0.5768**	0.5843	1.2835	0.5766	0.0347
8	**0.6193**	0.6305	1.7763	0.6223	0.4844
9	**0.7599**	0.7732	1.7201	0.7593	0.0789
10	**0.9492**	0.9673	1.8712	0.9513	0.2212

TABLE 5. Comparison between FORTRAN Sol. Before, After Static and Dynamic Condensations by remaining 80**%** D.O.F.

MODE	Frequencies (cps)				
	Exact Sol.by FORTRAN	Static condensation by remaining80% D.O. F	Error %	Dynamic condensation by remaining 80%D.O.F	Error %
1	**0.0355**	0.0357	0.5602	0.0355	0.000
2	**0.0528**	0.0532	0.7519	0.0528	0.000
3	**0.2087**	0.2093	0.2867	0.2087	0.000
4	**0.2375**	0.2388	0.5444	0.2375	0.000
5	**0.3867**	0.3882	0.3864	0.3867	0.000
6	**0.4681**	0.4746	1.3696	0.4682	0.0214
7	**0.5768**	0.5845	1.3174	0.5762	0.1040
8	**0.6193**	0.6317	1.9629	0.6215	0.3552
9	**0.7599**	0.7802	2.6020	0.7633	0.4474
10	**0.9492**	0.9779	2.9347	0.9531	0.4108

TABLE 6. Comparison between FORTRAN Sol. Before, After Static and Dynamic Condensations by remaining 70% D.O.F.

MODE	Frequencies (cps)				
	Exact Sol.by FORTRAN	Static condensation by remaining70% DOF	Error%	Dynamic condensation by remaining 70%DOF	Error%
1	**0.0355**	0.0358	0.838	0.0355	0.000
2	**0.0528**	0.0533	0.9381	0.0528	0.000
3	**0.2087**	0.2113	1.2305	0.2087	0.000
4	**0.2375**	0.2398	0.9591	0.2375	0.000
5	**0.3867**	0.3887	0.5145	0.3867	0.000
6	**0.4681**	0.4749	1.4318	0.4683	0.0427
7	**0.5768**	0.5851	1.4186	0.5765	0.0520
8	**0.6193**	0.6322	2.0405	0.622	0.4359
9	**0.7599**	0.7833	2.9874	0.7653	0.7106
10	**0.9492**	0.9803	3.1724	0.9554	0.6532

TABLE 7. Comparison between FORTRAN Sol. Before, After Static and Dynamic Condensations by remaining 60% D.O.F.

MODE	Frequencies (cps)				
	Exact Sol.by FORTRAN	Static condensation by remaining 60% D.O. F	Error%	Dynamic condensation by remaining 60% D.O. F	Error%
1	**0.0355**	0.0361	1.6620	0.0355	0.000
2	**0.0528**	0.0537	1.6759	0.0528	0.000
3	**0.2087**	0.2124	1.7420	0.2087	0.000
4	**0.2375**	0.2401	1.0829	0.2375	0.000
5	**0.3867**	0.3943	1.9275	0.3868	0.0259
6	**0.4681**	0.4821	2.9039	0.4684	0.0641
7	**0.5768**	0.5862	1.6035	0.5766	0.0347
8	**0.6193**	0.6322	2.0405	0.6223	0.4844
9	**0.7599**	0.7854	3.2468	0.7657	0.7633
10	**0.9492**	0.9821	3.3499	0.9562	0.7375

TABLE 8. Comparison between FORTRAN Sol. Before, After Static and Dynamic Condensations by remaining 50% D.O.F.

ODE	Exact Sol.by FORTRAN	Static condensation by remaining50% D.O. F	Error%	Dynamic condensation by remaining 50%D.O. F	Error%
1	**0.0355**	0.0363	2.2034	0.0355	0.000
2	**0.0528**	0.0541	2.4029	0.0528	0.000
3	**0.2087**	0.2132	2.1107	0.2087	0.000
4	**0.2375**	0.2413	1.5748	0.2375	0.000
5	**0.3867**	0.3948	2.0517	0.3865	0.0517
6	**0.4681**	0.4833	3.1450	0.4678	0.0641
7	**0.5768**	0.5867	1.6874	0.5767	0.0173
8	**0.6193**	0.6329	2.1488	0.6224	0.5006
9	**0.7599**	0.7858	3.2960	0.7659	0.7896
10	**0.9492**	0.9827	3.4089	0.9569	0.8112

TABLE 9. Comparison between FORTRAN Sol. Before, After Static and Dynamic Condensations by remaining 40% D.O.F.

MODE	Frequencies (cps)				
	Exact Sol.by FORTRAN	Static condensation by remaining40%D.O. F	Error%	Dynamic condensation by remaining 40% D.O. F	Error%
1	**0.0355**	0.0366	3.0055	0.0355	0.000
2	**0.0528**	0.0547	3.4735	0.0528	0.000
3	**0.2087**	0.2145	2.7039	0.2087	0.000
4	**0.2375**	0.2532	6.2006	0.2376	0.0421
5	**0.3867**	0.3968	2.5453	0.3864	0.0776
6	**0.4681**	0.4844	3.3649	0.468	0.0214
7	**0.5768**	0.5886	2.0048	0.5769	0.0173
8	**0.6193**	0.6334	2.2261	0.6226	0.5328
9	**0.7599**	0.7875	3.5048	0.766	0.8027
10	**0.9492**	0.9933	4.4397	0.9571	0.8323

Example 4:

For the application of static and dynamic condensation on high-rise buildings, a three-dimensional building with a steel rigid frame system to resist gravity load as the main part and the lateral load as a secondary part was chosen, and it was analyzed a free analysis and forced analysis under seismic loads, (EL CENTRO) earthquake was chosen, and the results were compared between static and dynamic condensation for free analysis and forced analysis**.** The results were validated before condensation with the SAP 2000 program.

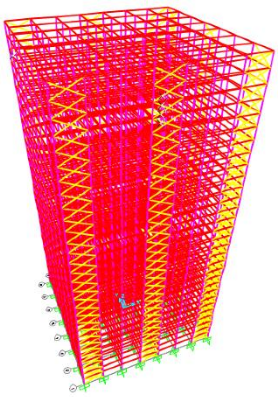

Fig.7 3D steel rigid frame.

The case study is dependent on a 30-story high-rise steel frame building with reinforced concrete slabs and brick walls used as a model as shown in Fig. (7). The rigid frames are arranged in X and Z directions of the horizontal square plane at distance of 7 meters of land with a square perimeter (7x6m) in the X-direction and the Z- direction, with a square area (42 x 42) m^2 with the height of the building of 90 meters, where it consisted of 30 stories and the height of the story was H= 3 m as shown in Fig. (8). The steel frames with a grade of steel S355 were used with Fy = 355MPa and Fu=480Mpa, used the beam section with sectors IPE (500) for inner or external beams but divided the steel columns into 5 groups, the 6 below stories were with sections H (400x990) and the next 6 stories with H (400x634), the next 6 with H (400x551), the next 6 with H (400x383), and the last 6 columns set with H (400x237). The steel bracing used to resist the lateral loads as a pipe of the steel with a radius R= 21.9cm and thickness t = 5mm. The solid slab is reinforced concrete with standard concrete grade C30/37 and it is a square slab with a cover weight = 0.15t/m^2. The mass of the building is from the D.L and 25% from the L.L as (Model Mode) in SAP2000. A FORTRAN program is Created to generate the required data file for SAP2000 and the FORTRAN program for static and dynamic condensation. The natural frequency (cps) results were calculated by the FORTRAN program before and after the static and the dynamic condensation 90, 80, 70, 60, 50, 40, 30, 20, and 10% respectively, by selecting the accurate master degrees of freedom and another time randomly selecting masters of DOFs. Then compared 1st 10-frequencies as shown in the Tables. (10), (11), (12), (13), (14), (15), (16), (17) & (18) between the natural frequencies before and after static and dynamic condensation.

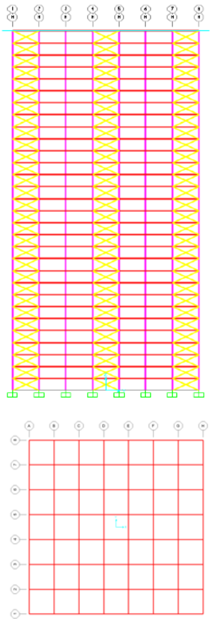

Fig.8 A square (42 x 42) m^2 with the height of 90 m.

A comparison between the choice accurately and the random selection of degrees of freedom, for example, when removing 40% of the degrees of freedom and preserving 60%, we find t

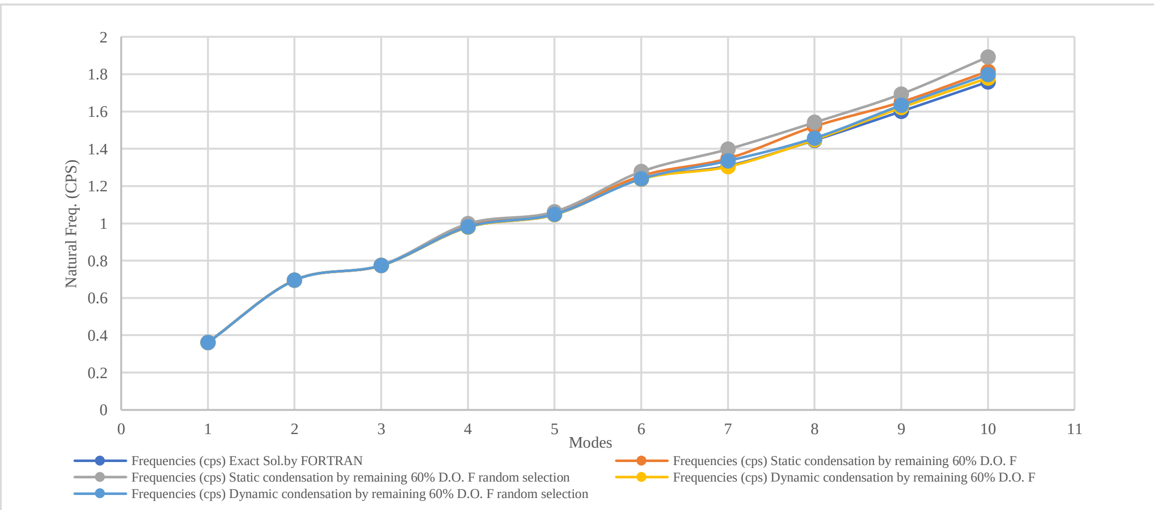

Fig. 9 Comparison between appropriate selection and random selection of DOFs by remaining 60 % DOFs.

Frequencies Results

TABLE 10. Comparison between FORTRAN Sol. Before, After Static and Dynamic Condensations by remaining 90% D.O.F

MODE	Frequencies (cps)				
	Exact Sol.by FORTRAN	Static condensation by remaining%90 D.O. F	Error%	Dynamic condensation by remaining %90D.O. F	Error%
1	**0.3620**	0.3620	0.000	0.3620	0.000
2	**0.6956**	0.6956	0.000	0.6956	0.000
3	**0.7741**	0.7741	0.000	0.7741	0.000
4	**0.9797**	0.9794	0.0306	0.9797	0.000
5	**1.0471**	1.0432	0.3738	1.0471	0.000
6	**1.2385**	1.2389	0.0242	1.2385	0.000
7	**1.3085**	1.3098	0.0993	1.3084	0.008
8	**1.446**	1.439	0.4864	1.444	0.138
9	**1.6008**	1.6235	1.3982	1.6104	0.5961
10	**1.7587**	1.8053	2.5648	1.7758	0.9516

TABLE 11. Comparison between FORTRAN Sol. Before, After Static and Dynamic Condensations by remaining 80% D.O.F.

MODE	Frequencies (cps)				
	Exact Sol.by FORTRAN	Static condensation by remaining80%D.O. F	Error%	Dynamic condensation by remaining 80%D.O. F	Error%
1	**0.3620**	0.3620	0.000	0.3620	0.000
2	**0.6956**	0.6956	0.000	0.6956	0.000
3	**0.7741**	0.7739	0.0258	0.7741	0.000
4	**0.9797**	0.9792	0.050	0.9797	0.000
5	**1.0471**	1.0422	0.4702	1.0471	0.000
6	**1.2385**	1.2402	0.1371	1.2385	0.000
7	**1.3085**	1.3125	0.3047	1.3086	0.007
8	**1.446**	1.454	0.5502	1.447	0.069
9	**1.6008**	1.6465	2.7756	1.6113	0.6559
10	**1.7587**	1.8141	3.0538	1.7823	1.342

TABLE 12. Comparison between FORTRAN Sol. Before, After Static and Dynamic Condensations by remaining 70% D.O.F.

MODE	Frequencies (cps)				
	Exact Sol.by FORTRAN	Static condensation by remaining 70% D.O. F	Error%	Dynamic condensation by remaining 70%D.O. F	Error%
1	**0.3620**	0.3620	0.000	0.3620	0.000
2	**0.6956**	0.6956	0.000	0.6956	0.000
3	**0.7741**	0.7738	0.0387	0.7741	0.000
4	**0.9797**	0.9834	0.3762	0.9797	0.000
5	**1.0471**	1.0543	0.6829	1.0471	0.000
6	**1.2385**	1.2498	0.9041	1.2385	0.000
7	**1.3085**	1.3475	2.8942	1.3093	0.0611
8	**1.446**	1.5167	4.6614	1.4484	0.1659
9	**1.6008**	1.6615	3.6533	1.6143	0.8433
10	**1.7587**	1.8141	3.0538	1.7851	1.5011

TABLE 13. Comparison between FORTRAN Sol. Before, After Static and Dynamic Condensations by remaining 60% D.O.F.

MODE	Frequencies (cps)				
	Exact Sol.by FORTRAN	Static condensation by remaining 60% D.O. F	Error%	Dynamic condensation by remaining 60% D.O. F	Error%
1	**0.3620**	0.3620	0.000	0.3620	0.000
2	**0.6956**	0.6956	0.000	0.6956	0.000
3	**0.7741**	0.7745	0.0387	0.7741	0.000
4	**0.9797**	0.9853	0.9994	0.9797	0.000
5	**1.0471**	1.0586	0.9943	1.0471	0.000
6	**1.2385**	1.2548	0.9891	1.2385	0.000
7	**1.3085**	1.3481	0.9870	1.3041	0.3363
8	**1.446**	1.5202	0.9706	1.4492	0.2213
9	**1.6008**	1.6509	0.9512	1.6211	1.2681
10	**1.7587**	1.8152	0.9696	1.7792	1.1656

TABLE 14. Comparison between FORTRAN Sol. Before, After Static and Dynamic Condensations by remaining 50% D.O.F.

MODE	Frequencies (cps)				
	Exact Sol.by FORTRAN	Static condensation by remaining50% D.O. F	Error%	Dynamic condensation by remaining 50%D.O. F	Error%
1	**0.3620**	0.3620	0.000	0.3620	0.000
2	**0.6956**	0.6956	0.000	0.6956	0.000
3	**0.7741**	0.7748	0.0903	0.7741	0.000
4	**0.9797**	0.9861	0.6490	0.9797	0.000
5	**1.0471**	1.0589	1.1144	1.0471	0.000
6	**1.2385**	1.2551	1.3226	1.2388	0.0242
7	**1.3085**	1.341	2.4235	1.3044	0.3133
8	**1.446**	1.5201	4.8746	1.4501	0.2335
9	**1.6008**	1.651	3.0405	1.6226	1.3618
10	**1.7587**	1.8154	3.1232	1.7801	1.2168

TABLE 15. Comparison between FORTRAN Sol. Before, After Static and Dynamic Condensations by remaining 40% D.O.F.

MODE	Frequencies (cps)				
	Exact Sol.by FORTRAN	Static condensation by remaining40%D.O. F	Error%	Dynamic condensation by remaining 40% D.O. F	Error%
1	**0.3620**	0.3620	0.000	0.3620	0.000
2	**0.6956**	0.6956	0.000	0.6956	0.000
3	**0.7741**	0.7751	0.1290	0.7741	0.000
4	**0.9797**	0.9865	0.6893	0.9797	0.000
5	**1.0471**	1.0591	1.1330	1.0473	0.0191
6	**1.2385**	1.2553	1.3383	1.2390	0.0404
7	**1.3085**	1.331	1.6904	1.3041	0.3363
8	**1.446**	1.5201	4.8747	1.4523	0.4357
9	**1.6008**	1.6567	3.3742	1.6231	1.3931
10	**1.7587**	1.8156	3.1339	1.7814	1.2907

TABLE 16. Comparison between FORTRAN Sol. Before, After Static and Dynamic Condensations by remaining 30**%** D.O.F.

MODE	Frequencies (cps)				
	Exact Sol.by FORTRAN	Static condensation by remaining30% D.O. F	Error%	Dynamic condensation by remaining 30%D.O. F	Error%
1	**0.3620**	0.3620	0.000	0.3620	0.000
2	**0.6956**	0.6955	0.0144	0.6956	0.000
3	**0.7741**	0.7756	0.1934	0.7741	0.000
4	**0.9797**	0.9887	0.9103	0.9797	0.000
5	**1.0471**	1.0601	1.2263	1.0477	0.0573
6	**1.2385**	1.2562	1.4090	1.235	0.2826
7	**1.3085**	1.3309	1.6831	1.3046	0.2980
8	**1.446**	1.5213	4.9497	1.4527	0.4633
9	**1.6008**	1.6571	3.3975	1.6238	1.4368
10	**1.7587**	1.8178	3.2512	1.7821	1.3305

TABLE 17. Comparison between FORTRAN Sol. Before, After Static and Dynamic Condensations by remaining 20**%** D.O.F.

MODE	Frequencies (cps)				
	Exact Sol.by FORTRAN	Static condensation by remaining20% D.O. F	Error%	Dynamic condensation by remaining 20%D.O. F	Error%
1	**0.3620**	0.3620	0.000	0.3620	0.000
2	**0.6956**	0.6951	0.0719	0.6956	0.000
3	**0.7741**	0.7761	0.2576	0.7741	0.000
4	**0.9797**	0.9851	0.5482	0.9797	0.000
5	**1.0471**	1.0626	1.4587	1.0479	0.0764
6	**1.2385**	1.2612	1.7999	1.238	0.0404
7	**1.3085**	1.3311	1.6978	1.3051	0.2598
8	**1.446**	1.5244	5.1430	1.4531	0.4910
9	**1.6008**	1.6689	4.0805	1.6247	1.4930
10	**1.7587**	1.8218	3.4636	1.7885	1.6944

TABLE 18. Comparison between FORTRAN Sol. Before, After Static and Dynamic Condensations by remaining 10**%** D.O.F.

MODE	Frequencies (cps)				
	Exact Sol.by FORTRAN	Static condensation by remaining10% D.O. F	Error%	Dynamic condensation by remaining 10%D.O. F	Error%
1	**0.3620**	0.3619	0.0276	0.3620	0.000
2	**0.6956**	0.6949	0.1007	0.6956	0.000
3	**0.7741**	0.7767	0.3347	0.7741	0.000
4	**0.9797**	0.9863	0.6692	0.9795	0.0204
5	**1.0471**	1.0635	1.5421	1.0484	0.1242
6	**1.2385**	1.2626	1.9088	1.2426	0.3310
7	**1.3085**	1.3348	1.9703	1.3049	0.2751
8	**1.446**	1.5271	5.3107	1.4832	2.5726
9	**1.6008**	1.6702	4.1552	1.6251	1.5179
10	**1.7587**	1.8223	3.4901	1.7891	1.7285

CONCLUSIONS

Free and forced vibration for dynamic problems by the static and dynamic condensation of matrices for high-rise buildings has shown many important points that any researcher would be attentive to before using condensation. Several results of the research can be summarized in the following points.

Whether the analysis was free or forced, the results agreed that dynamic condensation is much more efficient in frequency results than static condensation, principally when solving dynamic problems, because dynamic condensation methods consider the inertia effects of DOF shapes that were ignored in static condensation. This is because the inertia is related to the inverse of the stiffness matrix, it cannot be obtained directly.

As the number and type of masters are chosen in degrees of freedom accurately, so the masters are selected must always be displacements rather than rotations, the selected master DOFs having the largest entries in the collective matrix, the DOFs having the largest movements in the patterns of interest, and the masters DOF that is associated with larger mass concentrations and is reasonably flexible relative to other mass concentrations and fixed constraints. Also, so is the number of master's degrees remaining or degrees of freedom removed, as influential to the validity and accuracy of the results. Where 10% is removed and 90% of the degrees of freedom are preserved, it is closer to accuracy than 20% is removed and 80% is preserved, i.e. The fewer the remaining degrees of freedom, the lower the accuracy.

Finally, it is emphasized that the dynamic condensation of dynamical problems gives a maximum deviation of ± 6: 7% if the selection of the master's degrees of freedom is accurate and appropriate from Sap 2000.

REFERENCES

[1] R. J. Guyan, "Reduction of stiffness and mass matrices," *AIAA Journal, American Institute of Aeronautics and Astronautics*, 1965.

[2] B. Irons, "Structural eigenvalue problems - elimination of unwanted variables", *American Institute of Aeronautics and Astronautics Journal 3 (5)* ,1965.

[3] A.Y.-T. Leung, "An Accurate Method of Dynamic Condensation in Structural Analysis," *International Journal of Numerical Methods in Engineering, Vol. 12, pp. 1705-1715*, 1978.

[4] V. N. Shah and M. Raymund, "Analytical selection of masters for the reduced eigenvalue problem.", International Journal of Numerical Methods in Engineering, Vol.18, 1982.

[5] M. Paz, "Dynamic condensation.", AIAA J., 22(5), 724-727, 1984.

[6] K. W. Matta, "Selection of Degrees of Freedom for Dynamic Analysis." Journal of Pressure Vessel Technology, Vol. 109/65, 1987.

[7] X. Lu, "Simplified Dynamic Condensation in multi-substructure systems". Canputers and Structures Vol 30. No 4, 1988.

[8] M. Paz, "Modified dynamic condensation method", *Journal of Structural Engineering, American Society of Chemical Engineers 115 (1) 234–238*. 1989.

[9] J.C. O'Callahan, "A new procedure for an improved reduced system (IRS) model", *Proceedings of the Seventh International Modal Analysis Conference*, 1989.

[10] N. Bouhaddi and R. Fillod, "A Method for Selecting Master DOF in Dynamic Substructuring using the Guyan Condensation Method.", *Canputers and Structures Vol.45, No.S/6, pp.94I-946*, 1992.

[11] J.H. Gordis, "An analysis of the improved reduced system (IRS) model reduction procedure", *Proceedings of the 10th International Modal Analysis Conference 1*, 1992.

[12] L. E. Suarez and M. P. Singht, "Dynamic Condensation Method for Structural Eigenvalue Analysis.", *AIAA JOURNAL Vol. 30, No. 4*, 1992.

[13] N. Zhang, “Dynamic Condensation of Mass and Stiffness Matrices.” Journal of Sound and Vibration, 188(4), pp. 601-615, 1995.

[14] S. Y. Chen and Y. G. Tsuei, “Estimation of System Matrices by Dynamic Condensation and Application to Structural Modification” AIAA JOURNAL Vol. 33, No. 11 , 1995.

[15] M .I. Friswell, S. D. Garvey, J .E. T. Penny, “Model reduction using dynamic and iterated IRS techniques.”, Journal of Sound and Vibration 186 (2), 1995.

[16] M. Naguib “Natural Frequencies and Modes of Vibrations for Cable Roofs”, Mansoura Engineering Journal (MEJ), Vol. 21, No. 2, June1996.

[17] K.-O. Kim, “Improved hybrid dynamic condensation for eigenproblems.”, American Institute of Aeronautics and Astronautics, Inc. AIAA Meeting Papers on Disc, 1996.

[18] Z. Hou and S. Chen, “Iterated Dynamic Condensation Technique and Its Applications in Modal Testing. ”, Shock and Vibration, Vol. 4, No.3, pp. 143-151, 1997.

[19] Z.-Q. Qu and Z.-F. Fu, “An Iterative method for Dynamic Condensation of Structural Matrices.”, available online at http://www.idealibrary.com, 1998.

[20] Z.-Q. Qu, “Structural Dynamic Condensation: Theory and Application” PhD Dissertation, State Key Laboratory of Vibration, Shock and Noise, Shanghai Jiao Tong University, Shanghai, PRC, 1998.

[21] M.I. Friswell, S.D. Garvey, J.E.T. Penny, “The convergence of the iterated IRS method.” Journal of Sound and Vibration 211 (1), 1998.

[22] N. Zhang and Q.S. Li, “Model Reduction of High-Rise Structures based on a Dynamic Condensation Method.”, Advances in Structural Engineering Vol. 2 No.4 /999, 1999.

[23] Z.-Q. Qu and W. Chan, “Dynamic condensation method for viscously damped vibration systems in engineering.” 1999.

[24] M.A. Rivera, M.P. Singh and L.E. Suarez, “Dynamic condensation approach for nonclassically damped structures.” *American Inst. of Aeronautics and Astronautics Journal 37 (5),* 1999.

[25] Z.-Q. Qu, Y. Shi, and H. Hua, “A reduced-order modeling technique for tall buildings with active tuned mass damper.” 2001.

[26] Z-Q Qu, R. P. Selvam, “Efficient method for dynamic condensation of nonclassically damped vibration systems.” *American Inst. of Aeronautics & Astronautics Jor.40(2),* 2002.

[27] G. V. Rao, “Dynamic Condensation and Synthesis of Unsymmetric Structural Systems.” Vol. 69, 2002.

[28] R. Lin, Y. Xia, “A new eigensolution of structures via dynamic condensation.” *Journal of Sound and Vibration 266,* 2003.

[29] Z.-Q. Qu, Y. Jung, R.P. Selvam, Model condensation for non-classically damped systems—part I: static condensation." *Mechanical Systems & Signal Processing 17 (5),* 2003.

[30] Z.-Q. Qu, R.P. Selvam, and Y. Jung., "Model condensation for non-classically damped systems—part II: iterative schemes for dynamic condensation.", *Mechanical Systems and Signal Processing 17 (5),* 2003.

[31] Y. Xia, and R. Lin, "Improvement on the iterated IRS method for structural eigensolutions.", *Journal of Sound and Vibration 270 (4–5),* 2004.

[32] M. Cho,and H. Kim, "Element-based node selection method for reduction of eigenvalue problems." American Institute of Aeronautics and Astronautics Journal 42 (8), 2004.

[33] H. Kim,and M. Cho, "Two-level scheme for selection of degrees of freedom and semi-analytic sensitivity based on the reduced system.", Computer Methods in Applied Mechanics and Engineering 195 (33–36), 2005.

[34] J. Boffa, N. Zhang and B. Samali, "Study on Model Reduction of Large Structural Systems for Active Vibration Control ." *Inst. of Materials Eng. Australasia Ltd*, 2005.

[35] P. Salvini,and F. Vivio, "Dynamic reduction strategies to extend modal analysis approach at higher frequencies." , Finite Elements in Analysis and Design 43, 2007.

[36] D. Choi, H. Kim, and M. Cho, "Iterative method for dynamic condensation combined with the substructuring scheme.", Journal of Sound and Vibration 317, 2008.

[37] K. Habgood and I. Arel, "A condensation-based application of Cramer's rule for solving large-scale linear systems.", Journal of Discrete Algorithms , 2011.

[38] A. Kaveh, H. Rahami, and H. Mehanpour, "Static and modal analyses of structures with different repeated patterns.", Advances in Engineering Software 51 , 2012.

[39] H.S. Kim and J.W. Kang, "An Efficient Structural Analysis of Super Tall Mega Frame Buildings Using a Multi-level Condensation Method", Journal of Asian Architecture and Building Engineering, November 2012.

[40] S. Weng, W. Tiana, H. Zhu, Y. Xia, F. Gao, Y. Zhang, and J. Li, "Dynamic condensation approach to the calculation of structural responses and response sensitivities.", Mechanical Systems and Signal Processing 88, 2016. [Online]. Available: http://dx.doi.org/10.1016/j.ymssp.2016.59.6.973

[41] N. M. Saeed and A. S.K. Kwan, "Displacement and force control of complex element structures by Matrix Condensation,", Structural Engineering and Mechanics, Vol. 59, No. 6 ,2016. [Online]. Available: http://dx.doi.org/10.12989/sem [Accessed: 25-Nov-2016].

[42] M. Mousavi and A. H. Gandomi, "A hybrid damage detection method using dynamic-reduction transformation matrix and modal force error," Engineering Structures 111 (2016) 425–434, 2016. [Online]. Available: http://dx.doi.org/10.1016/j. engstruct.2015.12.033.

[43] N. Madutujuh, “Static Condensation of Building Models using Unit Load and Cholesky Matrix Inversion Method (Flexibility Method),”, 2017.

[44] S.-H. Boo and P.-S. Lee, “A dynamic condensation method using algebraic substructuring .” *Int. J. Numer. Meth. Engng; 109:1701–1720, 2017.* [Online]. Available: https://wileyonlinelibrary.com DOI: 10.1002/nme.5349. [Accessed: 02-Sep-2017].

[45] D. Li, “Discussion of model reduction and reservation.” *Procedia Eng.188 354–361*, 2017.

[46] A. Esmaeilzad and K. Khanlari, “Dynamic condensation of non-classically damped structures using the method of Maclaurin expansion of the frequency response function in Laplace domain,” *Journal of Sound and Vibration 426 (2018) 111e128* , 2018. [Online]. Available: https://doi.org/10.1016/j.jsv.2018.04.016

[47] A. Cammarata, R. Sinatra, and· P. D. Maddìo, “Static condensation method for the reduced dynamic modeling of mechanisms and structures. ”, 2019. [Online]. Available: https://doi.org/10.1007/s00419-019-01560-x [Accessed: 15-May-2019].

[48] W. Tian, S. Weng, Q. Xia, and Y. Xia, “Dynamic condensation approach for response-based finite element model updating of large-scale structures.” *Journal of Sound and Vibration 506 (2021) 116176, 2021.* [Online]. Available: https://doi.org/10.1016/j.jsv.2021.116176 [Accessed: 30-April-2021].

[49] R. I. Munteanu, F. Mota, V. Calofir and C. Baciu, “New Approach to Nonlinear Dynamic Analysis of Reinforced Concrete 3D Frames; An Accurate and Computational Efficient Mathematical Model.” 2022.

[50] M. Paz, W. Leigh, “Structural Dynamics Theory and Computation” 5th Edition.”2004.

[51] F. Fu, “Advanced Modelling Techniques in Structural Design.” 2015.

[52] Z-Q. Qu, “Model Order Reduction Techniques: with applications in finite element analysis. ” *1st edition, Originally published by Springer-Verlag London Berlin Heidelberg, 978-1-84996-924-6,* 2004

[53] L. Meirovitch, “Computational methods in structural dynamics.”, 1980 by Sijthoff & Noordhof International Publishers B.V, Alphen aan denRijn, The Netherlands.

[54] W. W. Jr and J. M. Gere, “Matrix Analysis of Framed Structures.” 3rd edition, 1990 by Van Nostrand Reinhold, ISBN-13: 978-1-4684-7489-3.

[55] F. R. Khan, “Recent Structural Systems in Steel for High-Rise Buildings.”, *In Proceedings of the British Constructional Steelwork Association Conference on Steel in Architecture*, 1969.

[56] B. S. Smith and A. Coull, “Tall building structures: analysis and design.”. 1991.

[56] B. Taranath, “Steel, concrete & composite design of tall buildings.” *New York: McGraw-Hill Book Company*, 1998.

[57] J. M. W. Brownjohn, T. C. Pan, and X. Y. Deng, “Correlating dynamic characteristics from field measurements and numerical analysis of a high-rise building.”, *Earthquake Engng Struct. Dyn. 2000; 29:523- 543,* 2000.

[58] M. Ali, “Evolution of concrete skyscrapers: from Ingalls to Jin Mao. Electronic Journal of Structural Engineering.”, *Electronic Journal of Structural Engineering, 1,* 2001.

[59] M. H. Gunel and H. E. Ilgin, “A proposal for the classification of structural systems of tall buildings.”, *Building and Environment 42 2667–2675,* 2007.

[60] E. Ibrahim, “High-Rise Buildings -Needs&Impacts.” *CIB world building congress,* 2007.

[62] M. M. Ali and K. S. Moon, “Structural Developments in Tall Buildings: Current Trends and Future Prospects.”, *Architectural Science Review Volume 50, Number 3,* Sep.2007.

[63] P. Jayachandran, “Design of Tall Buildings Preliminary Design & Optimization .” 2009.

[64] M. Elnimeiri and A. Almusharaf, “Structure and Architectural Form of Tall Buildings.” 2010.

[65] M. M. Ali and K. Al-Kodmany, “Tall Buildings and Urban Habitat of the 21st Century: A Global Perspective.” *Buildings 2012, 2, 384-423,* 2012.

[66] B. Zhao, J. He, X. Lu, B. Zhou and L. Yu, “On Seismic Design of Composite High-rise Buildings with SRC Column, Steel Beam and RC Core Tube.”, 2012.

[67] B. K. Nguyen and H. Altan, “Tall-Building Projects Sustainability Indicator (TPSI): A New Design and Environmental Assessment Tool for Tall Buildings.”, *Buildings,2,43-62* 2012.

[68] S. Cammarano, G. Lacidogna and A. Carpinteri, “Structural analysis of high-rise buildings under horizontal loads: A study on the Intesa Sanpaolo Tower in Turin.” 2013.

[69] K. A. Zalka, “Structural analysis of regular multi-storey buildings.”, *by Taylor & Francis Group, LLC,* 2013.

[70] K. Kayvani, “Design of high-rise buildings: past, present, and future.” , 23rd Australasian Conference on the Mechanics of Structures and Materials (ACMSM23) Byron Bay, Australia, 9-12 December 2014, S.T. Smith (Ed.), 2014.

[71] M. Alaghmandan, P. Bahrami, M. Elnimeiri, “The Future Trend of Architectural Form and Structural System in High-Rise Buildings .”, *Architecture Research, 4(3): 55-62,* 2014.

[72] L. Peng, “Form Follows Function–The Composite construction and Mixed Structures in Modern Tall Buildings. ” 2014.

[73] G. L. Lau, “Sustainable High-rise Construction in Shanghai: (Case study: Shanghai Tower).” July 2015.

[74] S. Y. Mousavi, "Sustainable high-rise building (Case study three example of a sustainable high-rise building in Iran).", *Int. Research Journal of Applied and Basic Sciences2251-838X /Vol,9(11),* [Online]. Available: http://www.irjabs.com. 2015.

[75] Y. J. Cha, P. Trocha and O Büyüköztürk, "Field Measurement-Based System Identification and Dynamic Response Prediction of a Unique MIT Building.", 2016.

[76] E. Hallebrand and W. Jakobsson, "Structural Design of High-Rise Building." 2016.

[77] J. Riad, "High-Rise Building Design, A design tool combining stakeholders and demands with design." 2016.

[78] I. Kovacevic and S. Dzidic "Modern Structural Concepts for High-Rise Building." 2018.

[79] K. Czynska, "A Brief History of Tall Buildings in The Context of cityscape Transformation in IN Europe.", *Space & FORM | przestrzeń i FORMa '36_2018,* 2018.

[80] A. A. Abd EL-Tawab, "Comparing the structural system of some contemporary high rise building form." 2018.

[81] M. M. Ali and K. S. Moon, "Advances in Structural Systems for Tall Buildings: Emerging Developments for Contemporary Urban Giants.", *Buildings 2018, 8, 104,* 2018.

[82] J.Y.R. Liew, Y.S. Chua and Z. Dai, "Steel concrete composite systems for modular construction of high-rise buildings." 2019.

[83] P. Ray, S. Roy, J. Qosimov, and O. Nurova, "Skyscrapers: Origin, History, Evolution and Future .", J. *Today's Ideas - Tomorrow's Technol., Vol. 6, No. 1, pp. 9-20*, 2019.

[84] S. Khudainazarov, T. Mavlanov, J. Qosimov, and O. Nurova, "Forced vibrations of high-rise buildings." 2020.

[85] S. D. Astarini and C. Utomo, "Performance-Based Building Design of High-Rise Residential Buildings in Indonesia." *Sustainability 2020, 12, 7103*, 2020.

[86] M. D. Meyer, "Wind and Earthquake Resistant Buildings, Structural Analysis and Design." 2003.

[87] M. F. Effendi, I. F. Ridzqo, and S. W. Dharmatanna, "An Overview of High-Rise Buildings in Jakarta since 1967 to 2020", *6th (ICSBE), Earth and Environmental Science 933 (2021) 012001,* 2020.

[88] I. S. Abbood, M. A. Jasim and S. S. Weli, "High Rise Buildings: Design, Analysis, and Safety–An Overview ." 2021.

[89] A. Coull, "Free vibrations of regular symmetrical shear wall buildings." *Building Sci 10:127–33*. 1975.

[90] G.Y. Wang, "Vibration of Building and Structures, Beijing Technology Science Press." *pp.168-178*,1978.

[91] T. Balendra, S. Swaddiwudhipong, and, S.T. Quek "Free vibration of asymmetric shear wall-frame buildings", *Earth. Eng. Struct. Dyn., 12(5), 629 650,1984.* [Online]. Available: https://doi.org/10.1002/eqe.4290120505

[92] L. Qiusheng, C. Hong, and L. Guiqing. "Analysis of Free Vibrations of tall building" *J. Eng. Mech. 1994.120:1861-1876*, 1994.

[93] Q.S. Li, J.Q. Fang, A.P. Jeary, "Free vibration analysis of multi-storey buildings with narrow rectangular plane configuration", *Engineering Structures 21, 507–518*, 1999.

[94] Q.S. Li, J.Q. Fang, A.P. Jeary, "Free Vibration Analysis of Cantilevered Tall Structures under Various Axial Loads", *Engineering Structures 22, 525–534*, 2000.

[95] Q.S. Li, "Exact solutions for free vibration of shear-type structures with arbitrary distribution of mass or stiffness." *J. Acoust. Soc. Am., Vol. 110, No. 4*, October 2001.

[96] K.A. Zalka, "A simplified method for calculation of natural frequencies of wall-frame Buildings", *Eng. Struct., 23(12), 1544-1555, 2001. [*Online]. Available: https://doi.org/10.1016/S0141-0296(01)00053-0

[97] K.A. Zalka, "Hand method for predicting the stability of regular buildings, using frequency measurements", *Struct. Des. Tall Special Build, 12(4), 273-281*. 2003. [Online]. Available: https://doi.org/10.1002/tal.221

[98] K.A. Zalka, "A simple method for the deflection analysis of tall-wall-frame building structures under horizontal load", *Struct. Des. Tall Special Build, 18(3), 291-31*, 2009. [Online]. Available: https://doi.org/10.1002/tal.410

[99] M.S Choi, "Free vibration analysis of plate structures using finite element transfer stiffness coefficient method." *J. Mech. Sci. Tech., 17(6), 805-815*, 2003.

[100] R. Rahgozar, H. Safari, and P. Kaviani , "Free vibration of tall buildings using Timoshenko beams with variable cross-section", *Str.Under Shock and Impact VIII,N*, 2004.

[101] K. B. Bozdoğan and Du. Öztürk, "AN APPROXIMATE METHOD FOR FREE VIBRATION ANALYSIS OF MULTI-BAY COUPLED SHEAR WALLS", *Mathematical and Computational Applications, Vol. 12, No. 1, pp. 41-50*, 2007

[102] Coull, Alexander, and Bishwanath Bose, "Simplified Analysis of Frame Tube Structures." *Journal of Structural Engineering, ASCE 101, No. 11*, 1975.

[103] Coull, Alexander, and A. K. Ahmed, "Deflections of Framed-Tube Structures." Journal of Structural Engineering, ASCE 104, No. 5, 1978.

[104] Q. Wang, Sturm–Liouville equation for free vibration of a tube-in-tube tall building, *Journal of Sound and Vibration 191 (3), 349–355*, 1996.

[105] Q. Wang, "Modified ODE-solver for vibration of tube-in-tube structures, Computer Methods in Applied Mechanics and Engineering" 129 (1996) 151–156.

[106] O. A. Pekau, L. Lin and Z. A. Zielinski, “Static and Dynamic Analysis of Tall Tube-In-Tube Structures by Finite Story Method”, *Eng. Structures, Vol.18, No.7, pp.515-527,* 1996.

[107] Wen-Hae Lee, “Free Vibration Analysis for Tube-In-Tube Tall Buildings”, *Journal of Sound and Vibration 303 287–304,* 2007.

[108] Y. Gong, “A New Free Vibration Analysis Method for Space Mega Frames of Super Tall Buildings.” *Published under licence by IOP Ltd, Materials Science and Eng. 10,* 2010

[109] R. Kamgar and R. Rahgozar, “A simple approximate method for free vibration analysis of framed tube structures.” *Struct. Design Tall Spec. Build*. 2010.

[110] R. Rahgozar, M. Malekinejad and M. R. Jahanshahi, “Free vibration analysis of coupled shear walls with axial force effects.”, *The IES Journal Part A: Civil & Structural Engineering, Vol. 4, No. 4, Nov. 2011*, 224–231, 2011.

[111] M. Malekinejad, R. Rahgozar “A simple analytic method for computing the natural frequencies and mode shapes of tall buildings”, *Appl. Math. Modell, doi: 10. 1016/j. apm. 2011.10.018.* 2011.

[112] K. B. Bozdogan, “Free Vibration Analysis of Wall-Frame Structures by Differential Quadrature Method.” *Int. Journal of Eng, and Applied Sciences (IJEAS), Vol.3, Issue 4(2011)7-20,* 2011.

[113] R. Kamgar, M.M. Saadatpour, “A Simple Mathematical Model for Free Vibration Analysis of Combined System Consisting of Framed Tube, Shear Core, Belt Truss and Outrigger System with Geometrical Discontinuities”, *Appl. Math. Modell, doi: 10.1016/j.apm.2011.12.029,* 2011.

[114] M. Malekinejad and R. Rahgozar, “Free vibration analysis of tall buildings with outrigger-belt truss system”, *Earthquakes and Structures, Vol. 2, No. 1,89-107*, 2011.

[115] X. Zhenyan, G. Yun, and G. Yaoqing, “Free Vibration Analysis for Step Tubular Structures of Tall Buildings Supported on Elastic Foundation Soil Applied Mechanics and Materials” *Vols. 166-169 pp 194-199,* 2012. [Online]. Available: http://doi:10.4028/www.scientific.net/AMM.

[116] M. Malekinejad and R. Rahgozar, “An analytical approach to free vibration analysis of multi-outrigger–belt truss-reinforced tall buildings”, *Struct. Design Tall Spec. Build. 22, 382–398,* 2013.

[117] K.B. Bozdogan and D. Ozturk, “Free Vibration Analysis of the Tube-in-Tube Tall Buildings with the Differential Transform Method.” *Structural Eng. Vol. 17 No. 9,* 2014.

[118] I. Venanzi and F. Ubertini, “Free Vibration Response of a Frame Structural Model Controlled by a Nonlinear Active Mass Driver System” *Hindawi Publishing Corporation Advances in Civil Engineering, Volume 2014, Article ID 745814, 11 pages*. 2014 [Online]. Available: http://dx.doi.org/10.1155/2014/745814

[119] Sunil, S. S. Asadi and S. R. K. Reddy, “Free Vibration Analysis of Multi-Storeyed Buildings resting on different Soil / Rock Media at Hyderabad, Telangana State,

India", *Indian Journal of Science and Technology, Vol 9(37), DOI: 10.17485/ijst/2016/v9i37/99827, Oct*. 2016.

[120] M. Malekinejad, R. Rahgozar, A, Malekinejad and P. Rahgozar, "A continuous–discrete approach for evaluation of natural frequencies and mode shapes of high-rise buildings", *Int J Adv Struct Eng 8:269–280*, 2016.

[121] P. Rahgozar, "Free Vibration of Tall Buildings using Energy Method and Hamilton's Principle*", Civil Engineering Journal Vol. 6, No. 5*, May, 2020.

[122] M. Babaei, Y. Mohammadi and A. Ghannadiasl, "Free Vibration and Frequency Calculation in Tall Buildings with Stepped Tube-in-Tube Systems", *International Journal of Acoustics and Vibration, Vol. 26, No. 2*, 2021. [Online]. Available: https://doi.org/10.20855/ijav.2021.26.21752

[123] O. Aksogan, H. M. Arslan and B. S. Choo, "Forced vibration analysis of stiffened coupled shear walls using continuous connection method", *Engineering Structures 25, pp. 499–506*, 2003.

[124] M. A. Hariri-Ardebili, H. R. Samani and M. Mirtaheri. "Free and Forced Vibration Analysis of an Infilled Steel Frame: Experimental, Numerical, and Analytical Methods", *Hindawi Publishing Corporation, Shock and Vibration Volume 2014, Article ID 439591*, 2014. [Online]. Available: http://dx.doi.org/10.1155/2014/439591

[125] S. Yousefianmoghadam, A. Stavridis, and B. Moaveni, "Comparative Study on Modal Identification of a 10 Story RC Structure Using Free, Ambient and Forced Vibration Data", The Society for Experimental Mechanics, Inc. 2017, *Dynamics of Civil Structures, Vol 2, Conference Proceedings of the Society for Experimental Mechanics Series, DOI 10.1007/978-3-319-54777-0_33*, [Online]. Available: https://www.researchgate.net/publication/318235540

[126] M. E. Cyril, N. Chukwuemeka, O. Festus, O. Felix, "Forced Vibration Response of Double-Bay Multi-Storey Building Frames with Joints of Infinite Rigidity", *Advances in Science, Technology and Engineering Systems Journal Vol. 2, No. 4, 68-77*, 2017. [Online]. Available: https://dx.doi.org/10.25046/aj020410

[127] S. Khudainazarov, T. Mavlanov, J. Qosimov and O. Nurova, "Forced vibrations of high-rise buildings", *Materials Science and Engineering 869, 052047 doi:10.1088/1757-899X/869/5/052047*, 2020. [Online]. Available: https://iopscience.iop.org/article/10.1088/1757-899X/869/5/052047