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INVESTIGATION OF THE WEB COMPACTNESS LIMITS FOR TAPERED STEEL MEMBER

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الملخص العربى:

تعد العناصر متغيرة العمق من اكثر العناصر المستخدمة في المنشات الحديدية وذلك لما توفره من تكلفة في الحديد المستخدم حيث تقل ابعاد القطاعات تدريجيا تبعا لتغير العزم الناتج علي المنشأ و نجد انه عند اختيار تخانة عصب العناصر متغيرة العمق يتم حسابها علي اساس البعد الاكبر للقطاع نظرا لقصور الكود المصري في دراسة حدود النحافة للعناصر متغيرة العمق مما يؤدي الي تكلفة اكبر. في هذا البحث يتم دراسة حدود النحافة للعناصر متغيرة العمق و امكانية تقليل تخانات العصب عن طريق انشاء نموذج عددي باستخدام طريقة العناصر المحددة و حساب اجهاد الانبعاج الحرج الحادث و ذلك باجراء دراسة بارامترية باستخدام المتغيرات المختلفة مثل اختلاف التحميل و اختلاف نسبة طول العنصر الي السمك للعناصر و اقتراح قيم مختلفة لحدود النحافة للوح العصب في العناصر الصلب متغيرة العمق.

الكلمات الدالة : حدود النحافة, العناصر متغيرة العمق, الانبعاج الحرج, تغيير الاجهادات علي العصب.

ABSTRACT:

Metal buildings are typically designed to withstand lateral forces, in their transverse direction, through steel moment resisting frames composed of web-tapered I-shaped beams. Web-tapered steel members, in which deeper cross-sections are used throughout the regions subjected to larger internal forces, are often used within steel constructions to obtain greater material economy. However, current structural steel design specifications' methods for assessing the stability of web tapered steel members are largely based on those developed for prismatic steel members, resulting in overly conservative estimates of their ultimate strengths. The main objective is to investigate and relax the compactness limits for web tapered members as there is no clear formula to calculate the thickness of tapered member and so the web thickness is calculated according to the bigger depth ignoring the tapering effect to meet the compactness limits, a parametric study has been conducted using a verified finite element model using Sap software package. Several 3D models were conducted using the finite element program sap to see the relation between the buckling stress under compression and

yield stress in web tapered members, different loading parameters. Geometrical parameters (H/t) web, normalized plate length (α) and tapering ratio (R) shall be investigated.

KEYWORDS: Slenderness limits, Critical buckling stress, Tapered steel member, Compactness limits, Stress gradient on web.

1- INTRODUCTION

Metal buildings are typically designed to withstand lateral forces , in their transverse direction, through steel moment resisting frames composed of web-tapered I-shaped beams (i.e., rafters and columns), herein called metal building frames (MBF. Because these structural systems under design loads experience moment gradients throughout each member, non-prismatic members are commonly used. Web-tapered steel members, in which deeper cross-sections are used throughout the regions subjected to larger internal forces, are often used within steel constructions to obtain greater material economy. However, current structural steel design specifications' methods for assessing the stability of web tapered steel members are largely based on those developed for prismatic steel members, resulting in overly conservative estimates of their ultimate strengths and As a result, the efficiency benefits gained through their use are limited. Tapered members are utilized in constructions for different reasons, including structural efficiency and aesthetic beauty and have been utilized extensively in buildings and bridges for more than 50 years. The main objectives of the present work are investigate and relax the compactness limits for web tapered members as there is no clear formula to choose the thickness of tapered member and so the web thickness is calculated according to the bigger depth ignoring the tapering effect to meet compactness limits.

2- LITERATURE REVIEW

There are many methods to study and design of tapered members that have variable depth and inertia by a number of authors. These works led to the conclusion that (equation 1) shows good results for tapered members with small inclination angles 10 degrees or less so for larger angles using this theory is not acceptable especially when analyzing local instability phenomena flange or web buckling. The analytical critical shear buckling stress is given by equation listed by

Timoshenko [1] $\sigma c = \frac{K\,\pi^2 E}{12(1-\mu^2)(\frac{h}{t})^2}$ (1)

Beam and beam-column elements were developed by several researchers to simulate the behaviour of tapered members including the effects of web tapering, large deformations, second-order effects, residual stresses, geometric imperfections, and plasticity spreading [2–5]. However, these elements do not account for local instabilities.

Experimental tests were performed on 1/3 scaled tapered portal frames [6] and full-scaled tapered columns, beams, and beam-columns [7] to investigate the seismic performance of portal frames and member's flexural buckling. However, these studies did not investigate the local web instabilities and focused on global member stability and emphasized the importance to study the local buckling of web tapered.

A wide finite element parametric study was conducted by A.H.Salem, [8] and also empirical formulas were presented to determine the ultimate axial capacity of tapered slender I-sections as given by equations. (2) to (4).

$$\frac{Po}{Py} = \frac{C}{\frac{H}{t}^{0.4} * \frac{Bf}{Tf}^{0.25}} \qquad (2)$$

Where H is the larger web plate width, *bf* and *tf* are the flange's width and thickness, respectively.

$$C = 16.95 + 1.15R - \frac{7.3}{R} \qquad (3)$$

Where *R* is the tapering ratio (larger to smaller width ratio)

$$\frac{Pu}{Po}^{n} + \frac{Pu}{Po}^{1/n} = 1 \qquad (4)$$

Where n = 20(R), $Po = Pu$ for case L/ry and Pe is Euler's buckling load of the smaller section of the tapered column. Due to post-buckling strength, the web can take extra loads until buckling, depending on the geometry. Despite the post buckling was early discovered in 1866, elastic buckling was used as a basis for the design due to simple formulas till Basler first adopted the post buckling and so AISC and AASHTO followed likewise.

AISC [9] and AASHTO [10] specifications followed Basler's procedure as it treated the web plate to be simply supported in which the juncture to be simple therefore it could be very conservative design.

Lee [11] using numerical finite element models introduced new formulas taking into consideration the restraining effect between web and flanges.

Mirambell [12] presented analytical formulation for calculating the critical buckling stress of slender tapered web panels using numerical model as shown in figure (1) taking into consideration the post buckling behavior and local buckling of web also the presence of flanges was taken into account.

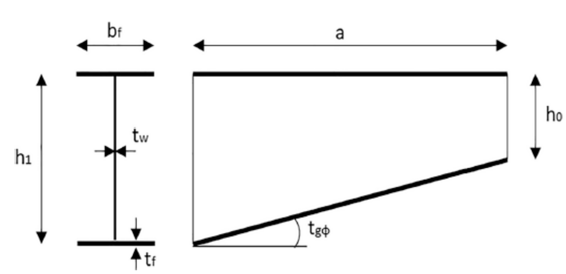


Figure 1: Geometric design parameters fo tapered plate girder

Another numerical investigation was conducted by Estrada [13] to present simple equations for shear buckling coefficient taking into account the real boundary conditions and material nonlinearity.

In 2014, new empirical approximate formulas for tapered girder FEM model as shown in figure (2) under shear and bending combined load to find the flexural-shear buckling resistance were introduced by Abu-Hamd [14] taking into account the effect of tapering angle) , the flange and web slenderness (λ). Four different types of load orientation, types 1, 3 moment increase shear resistance and types 2, 4 reduce shear resistance. By using finite element model, buckling multiplier (β) from buckling analysis was calculated and so $F_{cr,M}$ and $F_{cr,Q}$ could be calculated.

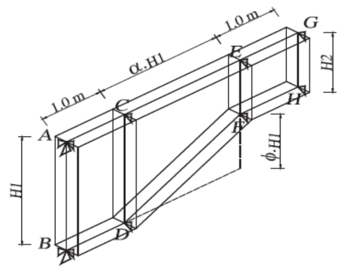


Figure 2: Tapered girder model

In 2020 an extensive finite element analysis was conducted by I.M.El Aghoury [15] to estimate the critical buckling factor for tapered members taking into account the boundary conditions (simple or fixed or restrained by flanges), tapering angle, the length over depth ratio and flange to web thickness ratio. Those loading parameters were taken uniform compression, pure shear and pure bending. New formulas were proposed to estimate the buckling coefficient for the three loading parameters and for the three different boundary conditions as follows: 1- for uniform compression

$$Katss = 4.295 + \frac{4.753}{\alpha^{0.853}} - \frac{0.573}{R^{2.646}} - \frac{4.474}{\alpha R} \qquad (5)$$

$$Katff = 6.936 + \frac{9.903}{\alpha^{0.879}} - \frac{9.266}{\alpha R} \qquad (6)$$

$$Katfr = Katss + \beta at(Katff - Katss) \qquad (7)$$

$$\beta at = 1.24\left(\frac{tf}{tw}\right) - 0.764 - 0.207\left(\frac{tf}{tw}\right)^2 \le 1 \qquad (8)$$

where K_{atSS} is axial buckling coefficient for tapered plate with simply supported edges, K_{atFF} is axial buckling coefficient for tapered plate with fixed edges, K_{atFR} is axial buckling coefficient for tapered plate with flange restricted edges, α is normalized plate length (a/H) and R is the tapering ratio (H/H1) which H is the large depth and H1 is the smallest depth and βat is a modification factor to account for the relative flange-to-web thickness.

3- FINITE ELEMENT MODEL AND MODEL VALIDATION

Several 3D models were conducted to investigate the relation between the buckling stress and yield stress in web of tapered members using Eigenvalue problem by finite element sap model. Loading parameters for different stress gradient, geometrical parameters (H/t) web, normalized plate length (α) which is (a/H) shall be investigated.

3.1 Model description

The typical studied configuration consists of tapered web plate represing simply supported web of a beam column with tapering ratio (R) which is the biggest depth at top to smallest width at bottom. The tapered plate is studied using different slenderness ratios "λ". Where λ is (H/t) and H is the bigger depth, t is the web thickness. The model configuration is shown in figure (3). The finite element model is built up using shell elements with six degrees of freedom at each node: translations in the x, y, and z directions, and rotations about the x, y, and z-axes.

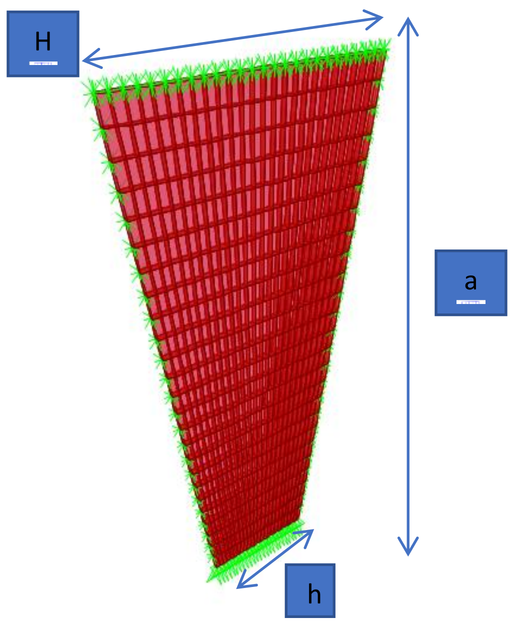


Figure 3: Overview of 3d model

3.2 Material propoerties

The following Table (3-1) summarizes the material properties used

Table 1: Material properties.

Weight	7.849	ton/m3
Modulus of Elasticity	21000000	ton/m2
Minimum yield stress (Fy)	24000	ton/m2

3.3 Boundary conditions

Figure (4) shows the boundary conditions for a typical 3d model through the study. As shown in figure, the lower edge (BC2) is restrained in translation x, y and z directions. All the other edges (BC1) are restrained in z direction (Perpendicular to the web plate) and free in x and y translation. Where x-y plane is the tapered web plate plane and z is the perpendicular axis to the web plate as shown in figure (3-2).

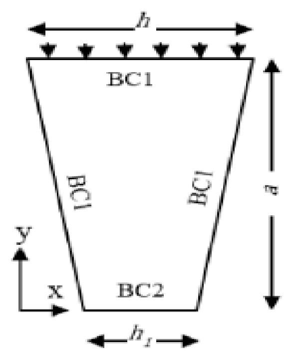
Ux represents translation in x direction.

Uy represents translation in y direction.

Uz represents translation in z direction.

F is for free translation

X is for restrained translation



	Ux	Uy	Uz
BC1	F	F	X
BC2	X	X	X

Figure 4: Boundary conditions for 3d model

3.4 Solution technique

Throughout this study, the finite element program sap [17] is used to run the Eigen value problem which is used to calculate the critical buckling stress for the plate element. Different parameters were used as illustrated before. For each model results of critical buckling were plotted against slenderness ratio.

3.5 Verification model

In 2020 an extensive finite element analysis was conducted by I.M.El Aghoury [15] to estimate the critical buckling factor for web tapered members taking into account the boundary conditions (simple or fixed or restrained by flanges), tapering angle, the length over depth ratio and flange to web thickness ratio. Those loading parameters were taken uniform compression, pure shear and pure bending. New formulas were proposed to estimate the buckling coefficient.

For verification, compression load is applied to the model for tapering ratio equals two for simply supported case and the results were compared to the proposed equation

$$Katss = 4.295 + \frac{4.753}{\alpha^{0.853}} - \frac{0.573}{R^{2.646}} - \frac{4.474}{\alpha R} \qquad (5)$$

A- 3d tapered column simply supported with web thickness of 26 mm and different aspect ratio (a/H) where a is the length of the member and H is the biggest depth and it ranges from 0.25 to 10, with tapering ratio R=H/h= 2 where H is the biggest depth and h is the smallest depth.

The models are shown in the following figure (5)

Uniform compression is applied to each model, the critical buckling coefficient is figured out through a numerical buckling analysis. Figure (6) shows the deformed shape of the lowest buckling mode of a uniformly compressed tapered plate under simply supported boundary conditions. Table 2 shows the results out of the models (axial buckling coefficient under compression) compared with K_{atss} proposed by I.M.El Aghoury [15] in equation 5.

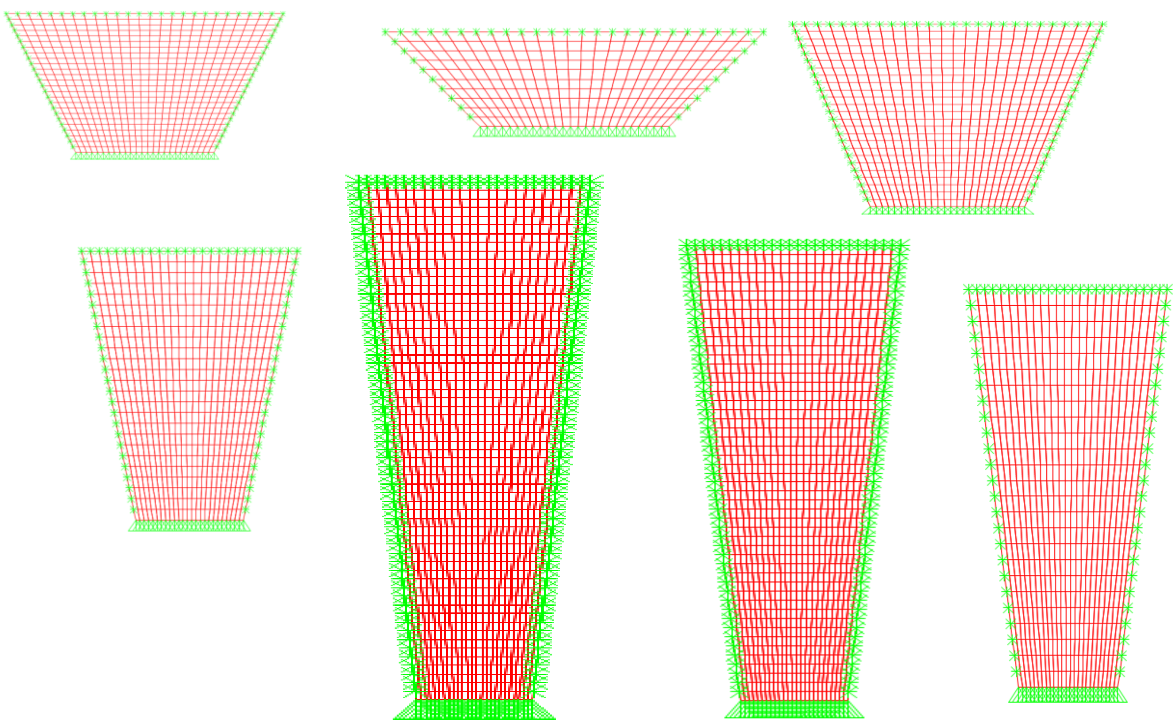


Figure 5: Typical finite element model with tapering ratio equals 2 and α ranges from 0.25 to 10

Table 2: lowest buckling mode factor (k)

α=A/H	Axial buckling coefficient K	Katss
0.25	22.5	10.734
0.5	18.09	8.286
0.75	9.9	7.268
1	7.57	6.691
1.25	6.31	6.315
1.5	6.01	6.047
1.75	5.87	5.846
2	5.75	5.688
2.25	5.61	5.561
2.5	5.52	5.456
2.75	5.46	5.367
3	5.4	5.292
4	5.2	5.073
10	4.97	4.618

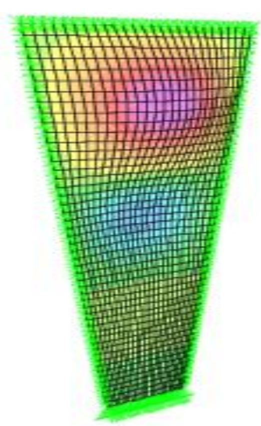


Figure 6: Typical deformed shape of lowest buckling mode of uniformly compressed plate under simple supported condition

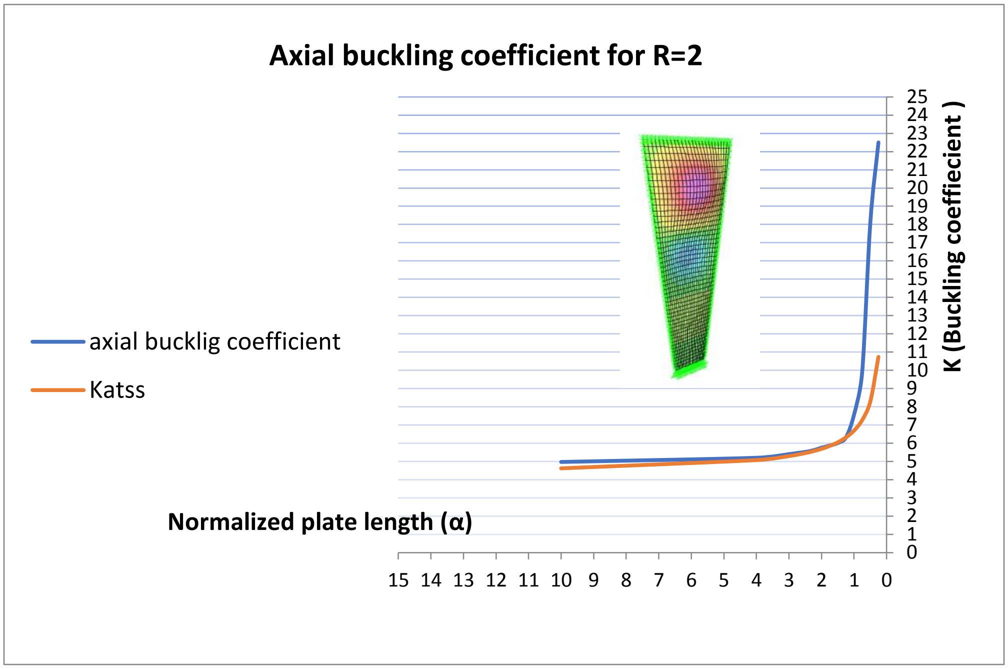


Figure 7: Axial buckling coefficient (k) vs α

Where α is the normalize plate length (a/H), (a) is the plate length an (H) is the biggest depth. As shown in figure (7), the axial buckling coefficient and the calculated katss in equation (2-10) calculated are identical except for $\alpha \leq 1$ as the typical buckling mode will not occur for these plates.

4- PARAMETRIC STUDY

4.1 Introduction

In this chapter, the verified finite element model discussed is adopted to investigate the slenderness limits for non-compact web tapered members under axial loading for different geometric and loading parameters as shown in table 3 and figure 8. The main studied parameters are

1- The normalized plate length α which is (a/H), (a) is the plate length and H is the bigger depth.
2- Different slenderness ratio by using different thickness.
3- Different loading parameters (N1/N2) which represent the stress gradient over the web tapered member for non-compact section.

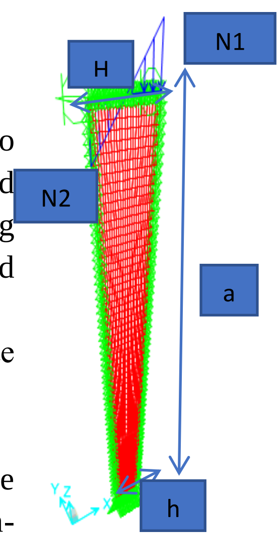


Figure 8: Studied parameters

4.2 Studied Parameters

As per Egyptian code [16], the web thickness of non-compact member can be calculated for rectangle section only and there is no formula for tapered members which results into conservative design.

However For each load case through the study, the initial web tapered plate thickness is calculated using table 3 in ECP. Then for each model the thickness shall be reduced gradually until the web tapered plate fails under buckling before reaching the yield stress. The following table (3) summarizes the different parameters for tapering ratio (R) = 2 Where R =H/h. H is the biggest depth and h is the small depth. **ψ**=ratio between tension stress and compression stress. For all models H = 100 cm, h is varying according to the tapering ratio (R) =2

Table 3: Different parameters

Parameters	Non-compact section		
Load gradient (N1/N2)	1	-1	0 & **ψ**=0
Normalized plate length (α)= a/H	3,4,6,8,10	3,4,6,8,10	3,4,6,8,10
Different thickness (t) mm	26,24,22,20,18	10,9,8,7,6	18,16,12,10,8

4.3 Analysis results

The results of the previously studied cases are shown in the following figures.

4.3.1 Non-compact web under compression

According to the Egyptian code [16] there is no formula to calculate the thickness of web tapered plate, however it could be calculated according to the compactness limits (width to thickness) ratio taking the biggest depth and thus more conservative results we shall get, so different thicknesses shall be used to calculate buckling stress for tapered members. The following equation shall be used $\frac{dw}{tw} = \frac{64}{\sqrt{fy}}$ (6)

For non-compact sections H= 100 cm, initial Web thickness = H*(fy^0.5)/64 = 2.42 cm. Different thicknesses 2.6, 2.4, 2.2, 2, 1.8 shall be used, also different plate length shall be used to find the critical point at which the plate buckles before reaching the yield stress. Normalized Buckling stress vs slenderness limits results are shown in the following figures

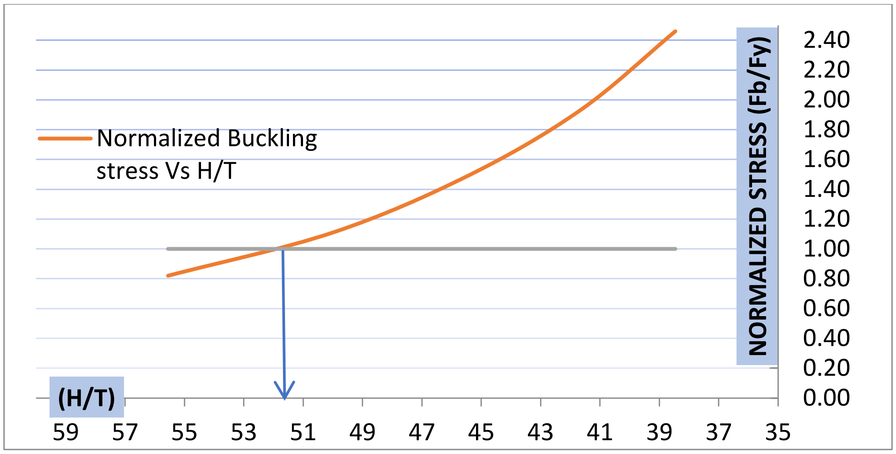


Figure 9: Normalized stress vs (H/t) for α=3 under compression stress gradient = 1

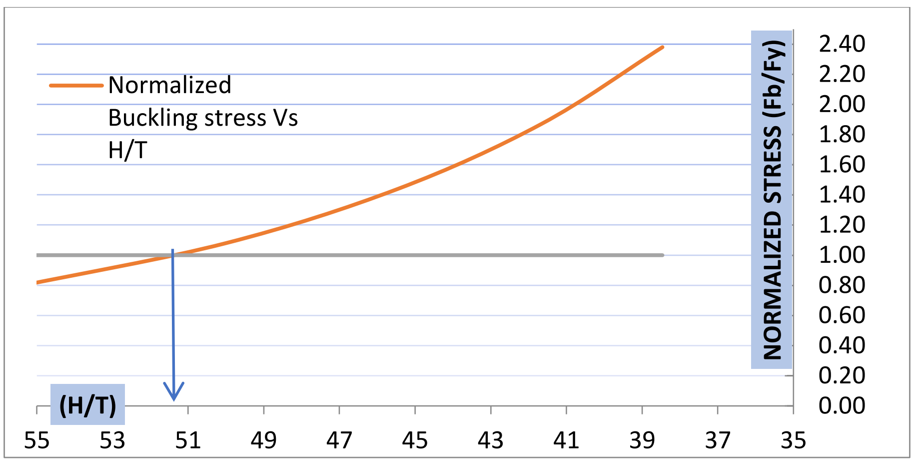


Figure 10: Normalized stress vs (H/t) for α=4 under compression stress gradient = 1

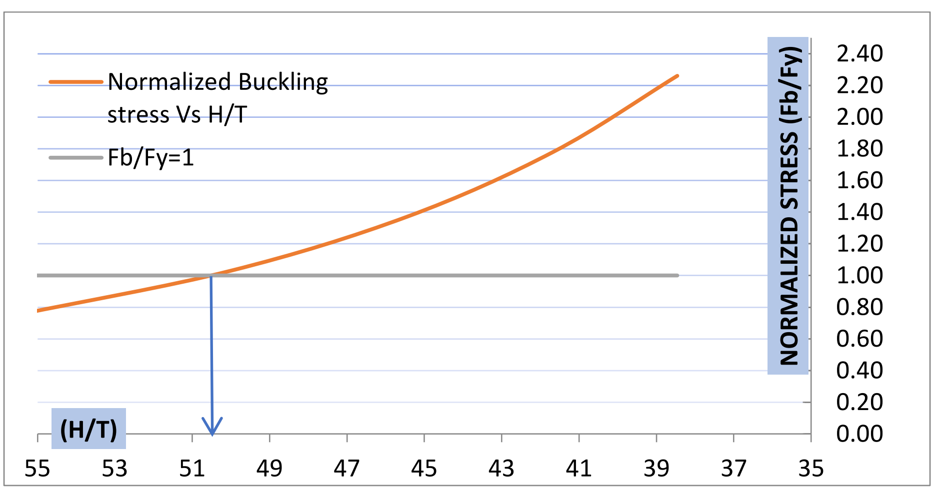


Figure 11: Normalized stress vs (H/t) for α=6 under compression stress gradient = 1

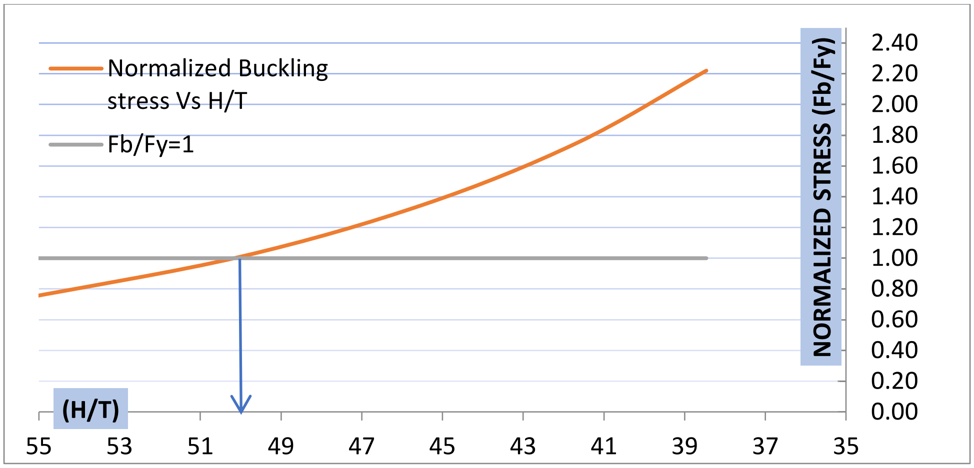


Figure 12: Normalized stress vs (H/t) for α=8 under compression stress gradient = 1

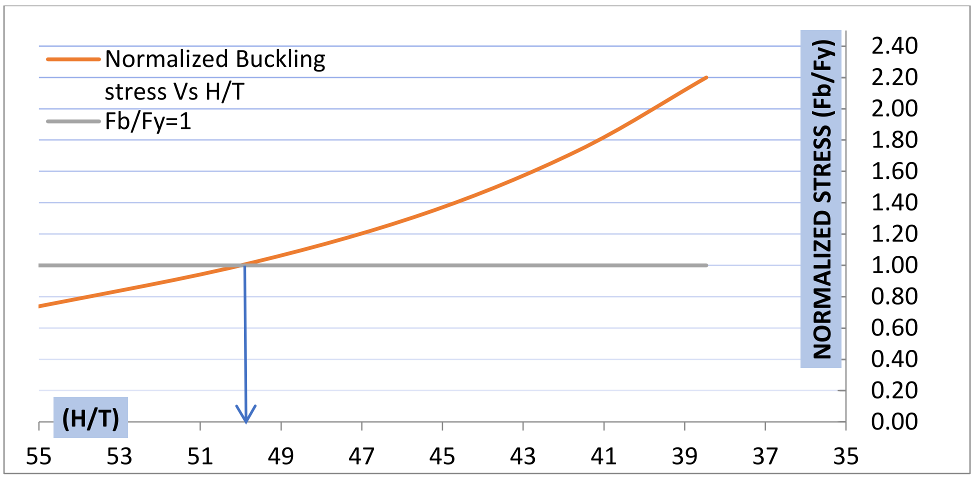


Figure 13: Normalized stress vs (H/t) for α=10 under compression stress gradient = 1

4.3.2 Non-compact web under bending

As per Egyptian code [16], there is no formula to calculate thickness of non-compact web tapered member, so thickness of non-compact web which is subjected to bending could be calculated using the following formula $\frac{dw}{tw} = \frac{190}{\sqrt{fy}}$ (7)

For Non compact web H= 100 cm,

Initial Web thickness = H*(fy^0.5)/190 = 0.82 cm.

Different thicknesses 1.0, 0.9, 0.8, 0.7, 0.6 shall be used ,also different plate length shall be used to find the critical point at which the plate buckle before reaching the yield stress.

Normalized Buckling stress vs slenderness limits results are shown in the following figures

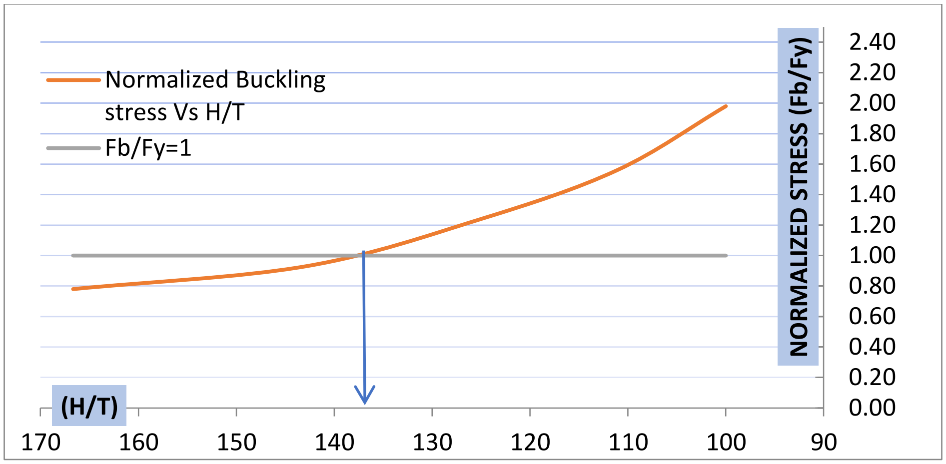


Figure 14: Normalized stress vs (H/t) for α=3 under bending stress gradient = -1

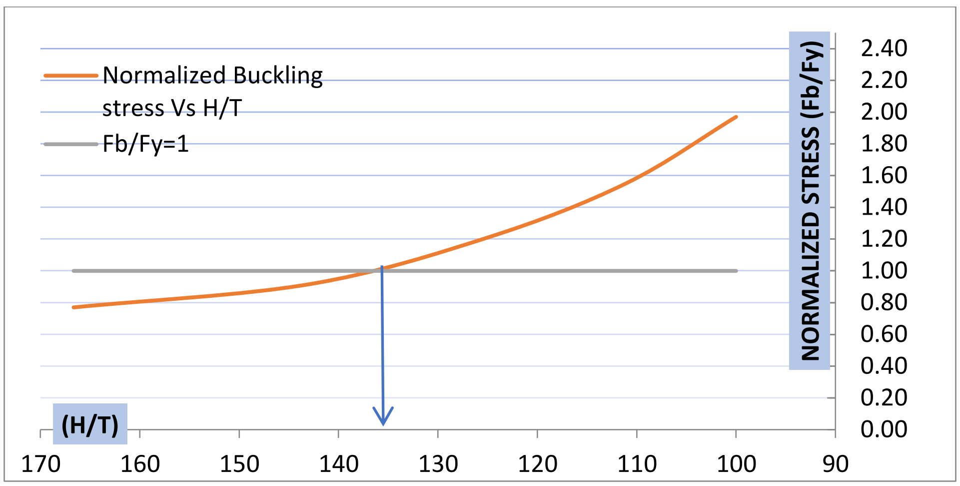


Figure 15: Normalized stress vs (H/t) for α=4 under bending stress gradient = -1

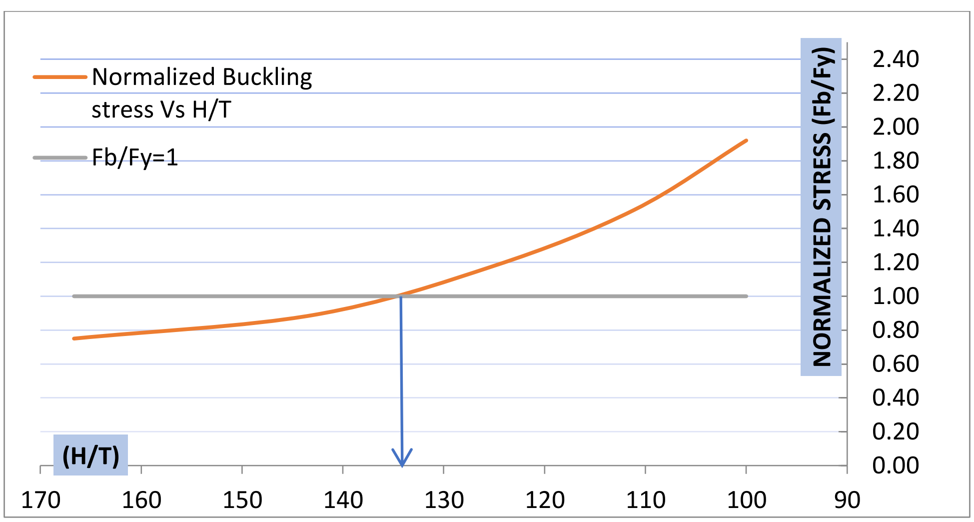


Figure 16: Normalized stress vs (H/t) for α=6 under bending stress gradient = -1

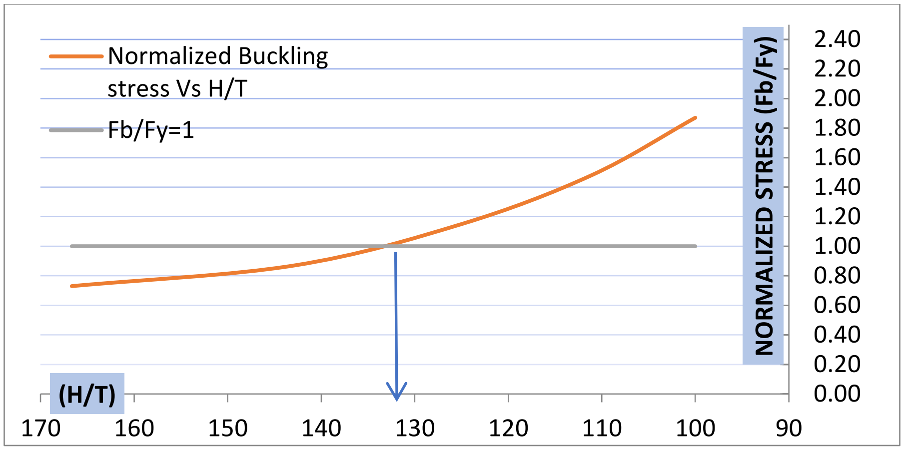


Figure 17: Normalized stress vs (H/t) for α=8 under bending stress gradient = -1

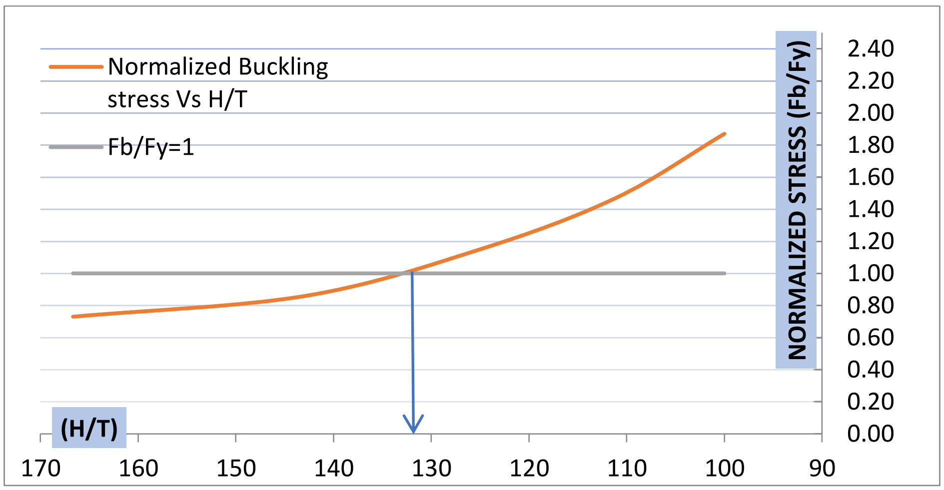


Figure 18: Normalized stress vs (H/t) for α=10 under bending stress gradient = -1

4.3. Non-compact web under bending and compression

As per Egyptian code [16], there is no formula to calculate thickness of non-compact web tapered member, so thickness of non-compact web which is subjected to bending and compression could be calculated using the following formula $\frac{dw}{tw} = \frac{190}{(2+\psi)\sqrt{fy}}$ (8) where (**ψ**) equals the ratio between tension stress and compression stress over the web, the case under study (**ψ**) equals zero.

For Non-compact web H= 100 cm, initial web thickness = H*(fy^0.5)*(2+0)/190 = 1.63 cm. Different thicknesses 1.8, 1.6, 1.4, 1.2, 1.0 shall be used ,also different plate length shall be used to find the critical point at which the plate buckle before reaching the yield stress. Normalized Buckling stress vs slenderness limits results are shown in the following figures.

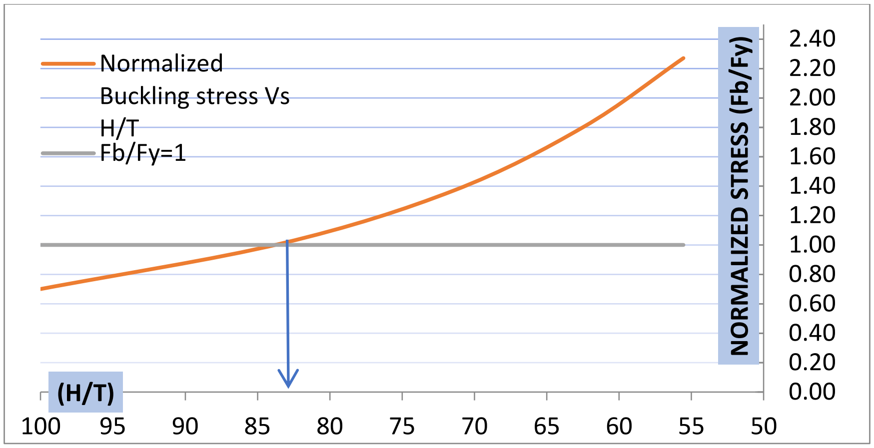


Figure 19: Normalized stress vs (H/t) for α=3 under compression and bending stress gradient = 0

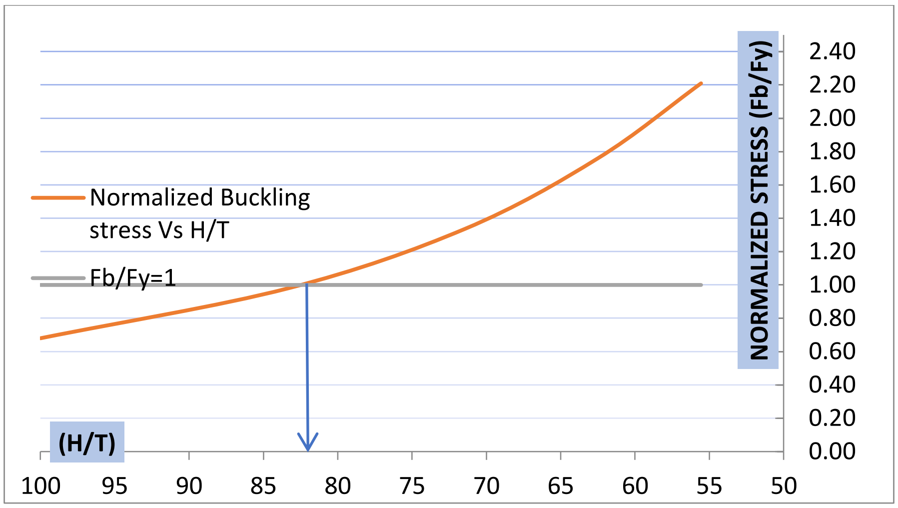


Figure 20: Normalized stress vs (H/t) for α=4 under compression and bending stress gradient = 0

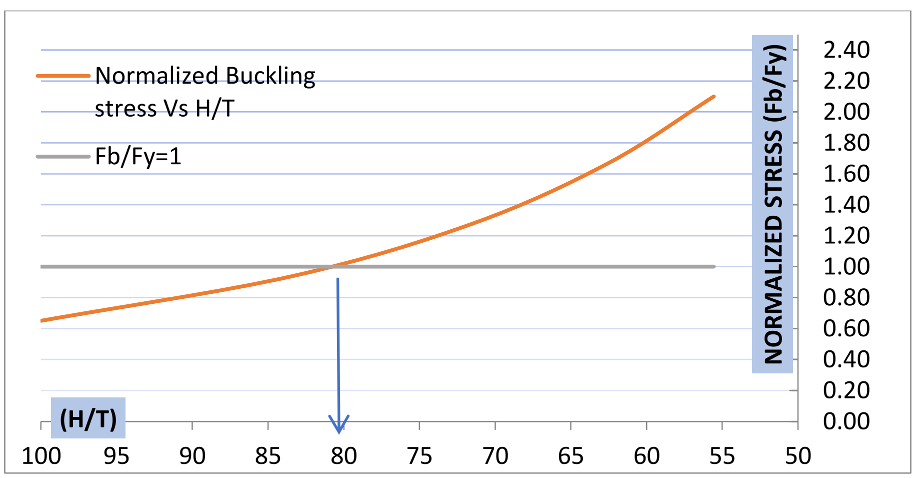


Figure 21: Normalized stress vs (H/t) for α=6 under compression and bending stress gradient = 0

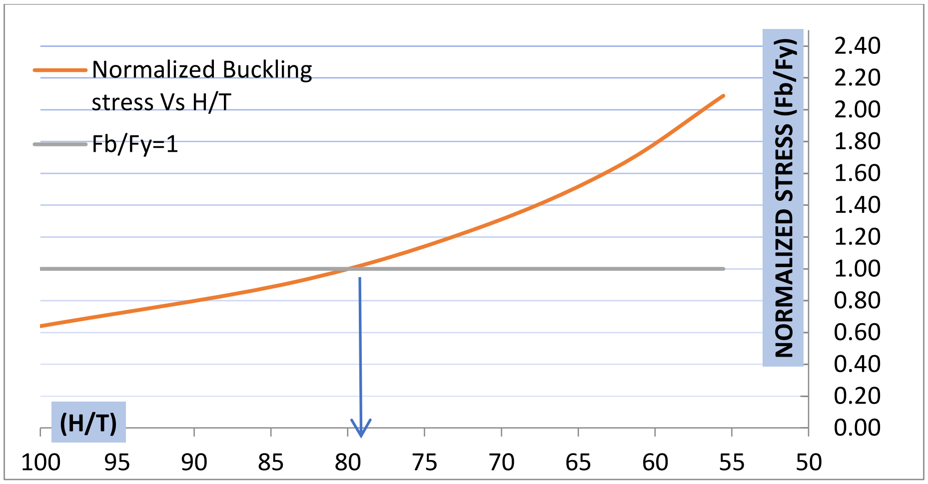


Figure 22: Normalized stress vs (H/t) for α=8 under compression and bending stress gradient = 0

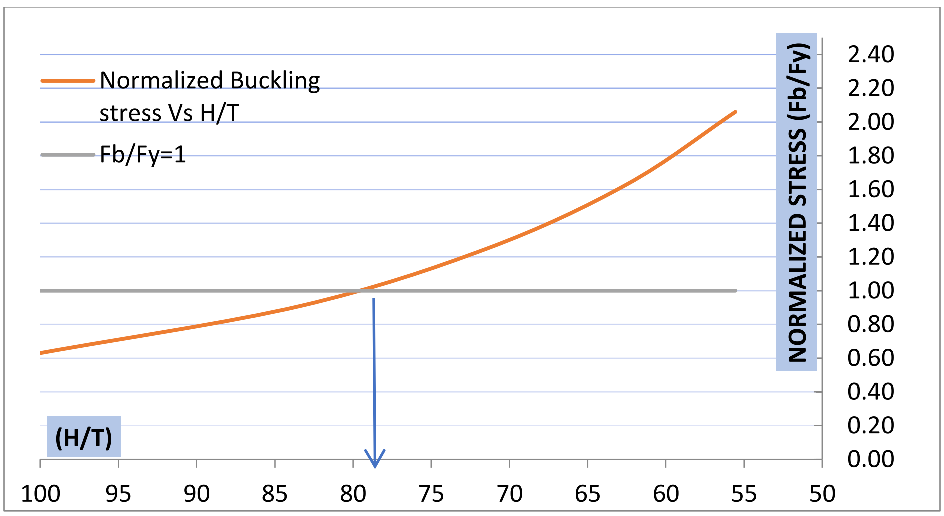


Figure 23: Normalized stress vs (H/t) for α=10 under compression and bending stress gradient = 0

4.4Discussion of the results

1. It can be noticed that for tapered members, compactness limits could be relaxed for all studied cases and so the code gives more conservative results.
2. It can be noticed that web thickness of tapered member under compression could be reduced from 24.2 mm to 20 mm by reduction (17%) for web tapered member and yet the section is still non-compact.
3. As tapered plate length (α) increases, normalized buckling stress decreases in non-compact web under compression.
4. It can be noticed that web thickness could be reduced from 8.1 mm to 7.5 mm by reduction (7%) for web tapered members under bending and yet the web is still non-compact.
5. As tapered plate length (α) increases, normalized buckling stress decreases for non-compact web under bending.

6. It can be noticed that web thickness could be reduced from 16.3 mm to 13 mm by reduction (20%) for web tapered members under compression and bending and yet the web is still non-compact.
7. As tapered plate length (α) increases the normalized buckling stress decreases for non-compact under bending and compression.

5- Summary and conclusion

An extensive finite element study is presented in this research to to find out whether the compactness limits of non-compact web tapered member could be reduced and yet preserve the section to be in the compactness limits. Finite element models are presented in this research to analyze the effect of different geometric parameters on the critical buckling stresses of web-tapered steel members. The elastic buckling coefficients are calculated for different stress gradient. The parameters included in this study are: the normalized plate length (α), different width to thicknesses (H/t). The parameters' ranges are selected to represent the extreme values for each parameter, yet keep it within the practical range of use and the specifications limitations when available. Three stress gradient types are included in the study (compression, bending, compression and bending)

The transvers edges of the plate are kept simply supported for all conditions. Normalized stress is calculated for all models.

5.1 Conclusions

The main conclusions of this study can be summarized as:

1. The existing compactness limits in the Egyptian code results into conservative thickness for tapered members and thus results into conservative design, however these limits could be relaxed as illustrated in this parametric study, yet keep the compactness limits and preserve the web to be non-compact according to the case under study.
2. Thickness of non-compact web of tapered members could be reduced.
3. As normalized plate length increases critical buckling stress decreases.

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