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For determination of critical slip surface slopes utilizing of Gray Wolf Optimization Technique

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الملخص العربى :

يعد استقرار المنحدر قضية مهمة في الهندسة الجيوتقنية. تستخدم طريقة التوازن الحدي (Limit Equilibrium Method) بشكل شائع لتحليل سطح الانزلاق الحرج بسبب بساطتها وفعاليتها. عادة ما يفترض أن سطح الانزلاق الحرج في طريقة التوازن الحدي ، والذي يحتوي على الحد الأدنى من معامل الأمان (FS) ، هو أفضل سطح منزلق ثم يستخدم كسطح انزلاقي في تحليل وتصميم تدابير تثبيت المنحدر ، مثل مسامير التربة والمراسي ، للمنحدرات غير المستقرة. تم استخدام العديد من التقنيات لتخصيص لتعين سطح الانزلاق الحرج في البعدين ، بينما قام البعض الآخر بحسابه في الأبعاد الثلاثة فقط ، ستقدم الدراسة الحالية كلا من السطحين المنزلقين للتربة باستخدام تقنية مستعمرة الذئاب الرمادية (GWO) بواسطة تطبيقه في العديد من أنواع المنحدرات المختلفة ثم المقارنة بين النتائج التي تم الحصول عليها من خلال تطبيق بعدين وثلاثة أبعاد لانواع مختلفة من المنحدرات.

ABSTRACT:

Slope stability is an important issue for geotechnical engineering, where many decades the limit equilibrium method (LEM) was commonly used to allocate the soil critical slip surface because it is too simple and effective. The (LEM) was used to determine the critical slip surface with the lowest factor of safety (FS), which is commonly thought to be the critical surface and utilized as the slip surface failure in the design of soil stabilizing measures such as anchors to safeguard unstable slopes. So Many techniques were used to allocate the critical slip surface in the two dimensions, while others calculated it by the three dimensions, but this paper will present the both two and three soil slip surfaces using the Gray Wolf Optimization technique (GWO), by applies it in many different types of slopes. The analysis will calculated by run a MATLAB program to gets the factors of safety results, then compared between the results obtained by both two and three dimensions for many different type of slopes.

Keyword: Slope stability, Factor of safety, Limit equilibrium, Gray wolf optimization

1. INTRODUCTION

A major part of geotechnical engineering practice is to study slope stability studies. The ability to analyze a mass of soil and determine a factor of safety (FOS) has provided the engineering profession with considerable credibility. Over several decades, improvements in the techniques used to analyze slopes have been tremendous and this has given rise to questions regarding the correct approach to be used in practice. In the early years of the soil mechanics, several slope stability methods developed. This often leaves questions about the important of the new methodologies to the practicing geotechnical engineer. Other more complicated computational tools have been born in the present, which used Two and three-dimensional analyses are used to analyze slope stability. This paper presented many Slope stability optimum solutions by using some artificial optimization techniques.

2. TWO-DIMENSION LIMIT EQUILIBRIUM METHODS

The most common methods for calculating the slope stability is the limit equilibrium techniques. Most limited equilibrium methods divide the mass into a series of vertical slices within a circular failure surface and analyze the forces inside the slopes. Limit equilibrium methods have achieved a big success in slope stability assessment under different conditions Fredlund (1981). The following equations present the factor of safety calculates by using the LEM. To calculate of the shear strength from the normal stress σ_n', the friction and the soil cohesion c' the following relationship is assumed

$$S = C' + \sigma_n' \tan\Phi'$$

According to Janbu (1954), the FOS defined as the ratio of the maximum shear strength S at failure to mobilized shear stress τ:

$$F = \frac{C' + \sigma_n' \tan\Phi'}{\tau} = \frac{S}{\tau}$$

2.1 Morgenstern and Price method

The Morg-Price (1965) method presented both the force & moment equilibrium. To define the direction of inter-slice normal force E and inter-slice shear force X with regard to the x direction, a function f(x) must be assumed. The values of λ each slice represent the various shear-to-normal ratios along the slip surface. The following Equation is presented, to evaluate slope stability which is based on an assumed that the ratio between the slice normal and shear forces.

$$X = E\,\lambda\,f(x)$$

The method suggests assuming any type of force function, for example half-sine, trapezoidal or user defined. Nash (1987) proved that the factor of safety F_f is equals to F_m in the following Equations:

$$F_f = \frac{\sum[\{\acute{c}\,l + (N - ul)\tan\acute{\emptyset}\}\sec\alpha]}{\sum\{W - (T_1 - T_2)\tan\alpha + \sum(E_2 - E_1)\}}$$

$$F_m = \frac{\sum[\{\acute{c}\,l + (N - ul)\tan\acute{\emptyset}\}]}{\sum\{W\sin\alpha\}}$$

2.2 Spencer Slice method

Spencer (1967) assumed perpendicular forces to the inter slice forces, derived two factors of safety equations one with respect to moment and the other is respect to forces as shown in the equations:

$$FS_m = \frac{\sum (C^{'}\beta R + (N - u\beta) R \tan \Phi^{'})}{\sum Wx - \sum Nf}$$

$$FS_f = \frac{\sum (C^{'}\beta \cos\alpha + (N - u\beta) \tan \Phi^{'} \cos\alpha)}{\sum (N \sin\alpha)}$$

Where: - N was calculated at each slice base as follows, u = pore pressure, R = circle radius, and α = inclination of slip surface at the middle of slice

$$N = \frac{\left(W - (X_R - X_L) - \frac{\sum C^{'}\beta \sin\alpha + uB \sin\alpha \tan\Phi^{'}}{FS}\right)}{\cos\alpha + \frac{\sin\alpha \tan\acute{\emptyset}}{Fs}}$$

3. THREE-DIMENSION LIMIT EQUILIBRIUM METHODS

Three-dimension critical slip surface can be solved by many Different numerical methods to simulate the stress field of a slope under natural conditions. The safety factor can be determined with respect to the moment of equilibrium that is equal to F_m, or calculated with respect to the force of equilibrium that is F_f. In terms of moment equilibrium Fm, the safety factor can be obtained by summing up the moment of all forces over the failed mass around a rotation axis. Similarly, by summing forces in the X direction over the failed mass, the safety factor with respect to force equilibrium Ff can be derived, the safety factors will be determined as follows

$$F_f = \frac{\tau^f}{\tau} = \frac{\sum_{i=1}^{n} (C_i + \sigma_i \tan \emptyset_i)}{\sum_{i=1}^{n} \tau_i}$$

$$F_m = \frac{M_R}{M_S} = \frac{\sum_{i=1}^{n} (C_i L_i + \sigma_i L_i \tan \emptyset_i)}{\sum_{i=1}^{n} \tau_i L_i}$$

4. GLOPAL OPTIMIZATION

The critical slip surface is a very complicated problem especially when taking the third direction into considerations, so many researcher applied different methods trying to reached to the optimum critical slip surface, this used methods used to solve the slope stability was classified into two main categories: 1) Methods based on mathematics, and 2) Meta-heuristics methods which also can classified into three main classes: physics-based techniques, Evolutionary algorithms (EA), and Swarm Intelligence algorithms (SI).

4.1 The mathematics methods

Yang, (2016) [15] Used finite element method to find the three-dimension critical slip surface and minimum factor of safety by using sphere and ellipsoid. An especially work for the critical slip surface and landslide volume under random earthquake ground motions was introduced by Huang, (2018). A novel approach to calculate the factor of safety of soil slopes when the soil strength follows the nonlinear yield criterion was presented by Zhou (2020).

4.2 Meta-heuristics methods

The physics-based techniques Mishra (2020) presented the application multiverse optimization algorithm (MVO) which applies the method on soil slopes containing a band of weak layers sandwiched between two strong layers, to determine if the proposed algorithm can capture the presence of the weak soil layer.

Evolutionary algorithms (EA) Mishra, (2020) presented teaching– learning-based optimization (TLBO). The effective modification for the gravitational search algorithm (MGSA) was shown in Khajehzadeh, (2012). A new mesh-free, particle-based numerical method in geotechnical engineering called smoothed particle hydrodynamic (SPH) is adopted to simulate the entire process of landslides, including the large displacement of soils after a landslide initiates was presented by Li, (2019).

Swarm Intelligence algorithms (SI) S. H. Li, (2020) applied the improved whale optimization algorithm (IWOA). Grey wolf optimization (GWO) which used to predict the soil slip surface Ali, at al. (2021). The Ant colony optimization presented on Dorigo, M. (2004), mimic the foraging behavior of social ants. Primarily, ants use pheromone as a chemical messenger and the pheromone concentration can also be considered as the indicator of quality solutions to a problem of interest. At the end Nagendran, (2020) presented the UAV technologies to calculate the critical slip surface. The Photogrammetry is a technique used for rock slope assessment.

5. MECHANSM OF GRAY WOLF TECHNIQUE

This work will present a Grey Wolf Optimization (GWO) which used to predict the soil slip surface by using MATLAB program. This optimization depends on simulated the hierarchy for the gray wolves consists of the following wolves (alpha, beta, delta, and omega) which focus on the wolves hunt strategy Mirjalili (2016).The leaders are known as alphas, and they might be male or female. The pack should heed his/her directives Mech, (1999). It is responsible for making hunting, sleeping place, ect, and these decisions are dictated to the other wolves which follows alpha. The key stages of grey wolf hunting are as follows, according to Muro, (2011)

1) Tracking, chasing and approaching the prey
2) Pursuing, encircling, and harassing the prey until it stops moving
3) Attack towards the prey

This paper will solve the soil problem by using the hunting strategy and the social hierarchy of grey wolves which mathematically modeled as follows.

5.1. Social hierarchy:

The social hierarchy of wolves in the mathematical model when developing GWO is considered the best approach to alpha (α), so, the second and third-best solutions will be beta (β) and delta (δ), so the remaining candidate solutions are omega (ω). Hunting (optimization) is guided by α, β, and δ. The ω wolves follow these three wolves

5.2. Encircling prey

Grey wolves encircle prey during the hunt, to mathematically model encircling behavior the following "Eq. (1), (2) are proposed:

$$\vec{\mathbf{D}} = \left|\vec{\mathbf{C}}.\overrightarrow{\mathbf{X}_p}(\mathbf{t}) - \vec{\mathbf{X}}(\mathbf{t})\right| \quad (1)$$

$$\vec{\mathbf{X}}(\mathbf{t}+\mathbf{1}) = \overrightarrow{\mathbf{X}_p}(\mathbf{t}) - \vec{\mathbf{A}}.\vec{\mathbf{D}} \quad (2)$$

Where t indicates the current iteration, A and C are coefficient vectors, X_p is the position vector of the prey, and indicates the position vector of a grey wolf. The vectors A and C are calculated by "Eq. (3), (4):

$$\vec{\mathbf{A}} = 2\vec{\mathbf{a}}\,.\overrightarrow{\mathbf{r_1}} - \vec{\mathbf{a}} \quad (3)$$

$$\vec{\mathbf{C}} = \mathbf{2}.\overrightarrow{\mathbf{r_2}} \quad (4)$$

Where components of a are reduced linearly from 2 to 0 throughout iterations and r_1, r_2 are random vectors in [0,1]. To see the effects of equations (1) and (2), A two and three-dimensional position vector and some of the possible neighbors are shown in Figure 1.a,b.

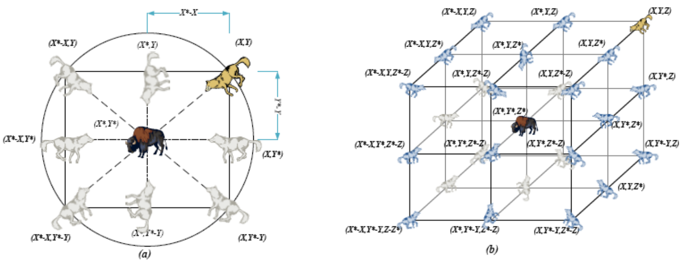


Figure 1 2D and 3D position vectors and their possible next locations

5.3. Hunting

Generally, the hunt is led by the alpha. The beta and delta could often also engage in hunting. However, there is no idea about the optimal position of the prey in a search area. To mathematically mimic the hunting behavior of grey wolves, the alpha was thought to be the best solution. Beta and delta are then supposed to provide a better understanding of the location of the prey. Thus, other studies such as omega will be saved and forced to change their positions according to the best search location.

5.4 Attacking prey (exploitation)

As mentioned above the grey wolves finish the hunt by attacking the prey when it stops moving. In order to mathematically model approaching the prey we decrease the value of $\vec{a}$. Note that the fluctuation range $\vec{A}$ of is also decreased by $\vec{a}$.In other words $\vec{A}$ is a random value in the interval [-a,a] where a is decreased from 2 to 0 over the course of iterations. When random values $\vec{A}$ of are in [-1,1], the next position of a search agent can be in any position between its current position and the position of the prey. Figure (2,a) shows that $|A|<1$ forces the wolves to attack towards the prey.

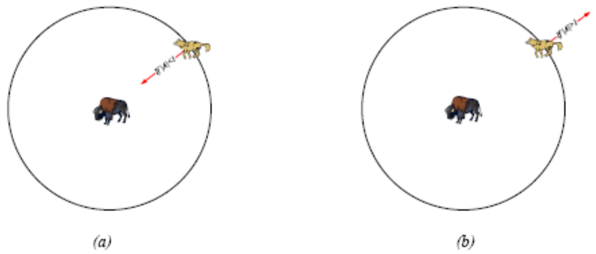


Figure 2 Attacking prey versus searching for prey

5.5 Search for prey (exploration)

Alpha, beta, and delta diverge from each other in real life to hunt for prey and unite to attack prey. So, a random value greater than 1 or less than -1 was used by the mathematical model to force the search agent to move far from the prey. Also, Figure (2.b) shows that in the case of $|A|>1$ this forces the grey wolves to diverge from the prey to hopefully find better prey. Another component that must take into consideration in GWO is $\vec{C}$. Which seen in "Eq. (4) the $\vec{C}$ vector contains random values from [0, 2].

6. INVISTIGATED SLOPES

With the rapid development of computer software, and optimization technique gradually becomes widely used for solving the complicated engineering problems. In this paper, MATLAB 2D and 3D were used for solving 4 different type of slopes (simple slope, homogenous slope, slope with benches, and steep slope), every slope will be solved using Spencer and Morg- Price methods under three different situations (soil without water, the water level is at 1.0 m under the soil surface, the water level at the middle of the soil slope), that will present 48 cases studies.

6.1 Simple uniform homogenous slope

The first case was a simple slope as shown in Figure (3.a), the parameters were shown in Table 1, and this slope case was used for the first time by Yamagami and Ueta (1997).

Table 1: the simple slope soil parameters (Yamagami and Ueta 1988)

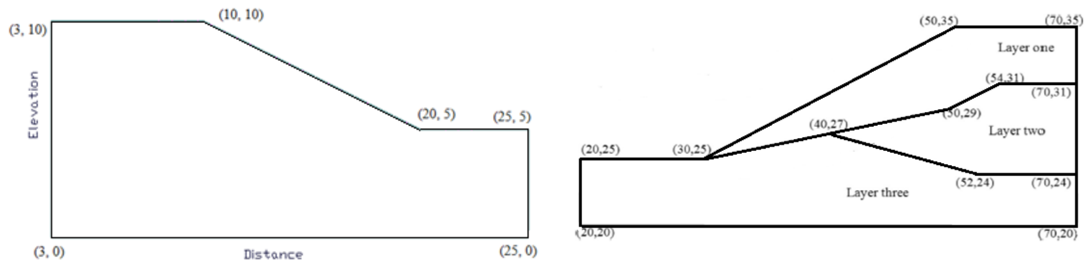
Soil	Soil type	Height t(m)	υ	γ kN/m³.	C(KPa)	Φ°
Layer one	Sand clay	10	0.3	17.64	9.8	10

6.2 Complicated slope

This slope have a many layers these consists of three layers of different materials presented by (Donald and Giam 1995), parameters of the three materials are tabulated on Table 2. Figure (3.b) shows the geometry of this slope.

Table 2: Parameters for each slope layers (Donald and Giam 1995)

Material	c (kN/m2)	friction angle Φ°	γ kN/m3
One	0.0	38.0	19.5
Two	5.3	23.0	19.5
Three	7.2	20.0	19.5



.(a) simple slope (b) Complicated heterogeneous

Figure 3 simple slope and Complicated heterogeneous slope geometries

6.3 Slope with benches

Figure (4.a) presented the soil slope consists of many benches with different inclinations as presented by Salah, and Yannic E, (2017). The Parameters for the slope layers are listed in Table 3.

Table 3: Parameters for slope layers (Salah.A, Yannic E, 2017)

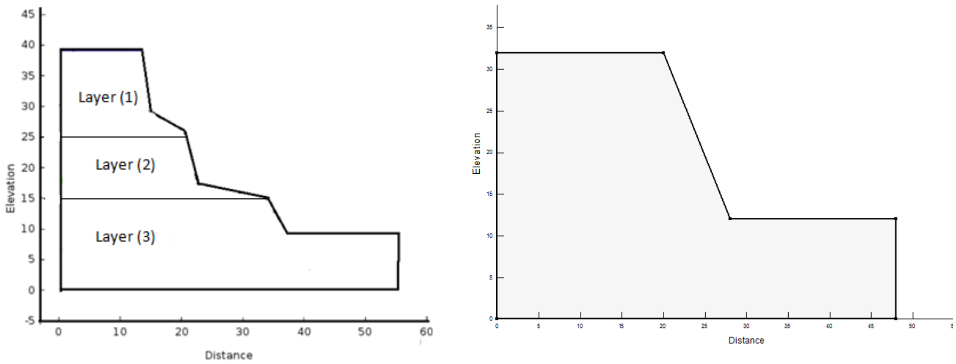
Material	Height (m)	c (kPa)	friction Φ°	γ kN/m3
Layer one	15	5	20°	18
Layer two	10	10	0	18.3
Layer three	15	5	20	17.5

6.4 Steep slope

The steep slope was presented by Salah, and Yannic E, (2017), characteristics of the soil layers are shown in Table 4 and it geometry will presented on figure (4.b).

Table 4: Parameters for steep slope layers (Salah.A, Yannic E, 2017)

Material	Height (m)	c (kPa)	friction angle Φ°	γ kN/m3
One layer	10	5	31o	18
Two layer	10	25	22 °	15.5
Three layer	10	5	31°	18



.(a) slope with penches (b) steep slope

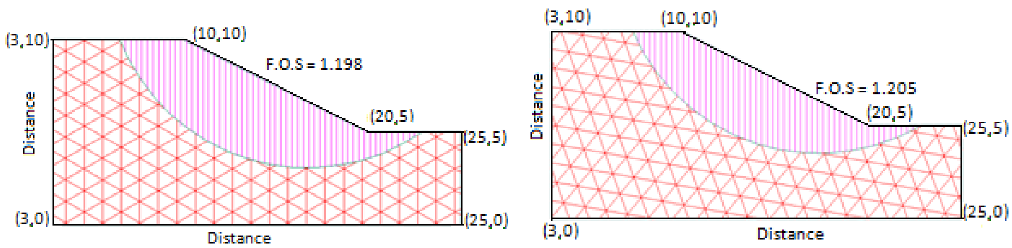
Figure 4 slope with penches and steep slope geometries

7. ANALYSIS, RESULTES, AND DISCUTION

As mentiened above this paper will applied the gray wolf optimization on four types of slopes on three different situations. In this section the simple slope analyses will be presented.

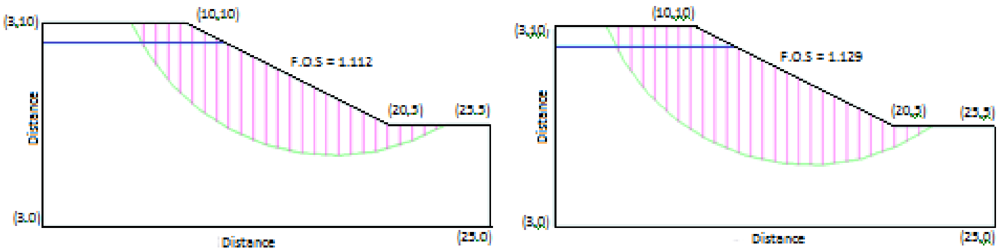
7.1 simple slope analysis and results

The 2D critical slip surface in case of dry soil is present in figure (5-a) computed using Morg–Price and its value was 1.198 and figure (5-b) presents the value of Spencer factor of safety which found to be 1.205. The optimum slip surfaces with its factors of safety shows in both figure (6- a, b) when using the same slope with 1.0 m water level below the slope surface found to be 1.112 and 1.129 by using Morg–Price and Spencer respectively. When the water level be in the middle of it the minimum FOS was found to be 1.178 in using the 2D method with Morg–Price method as shown in figure (7-a), and its value be 1.193 in Spencer method as shown in figure (7-b).



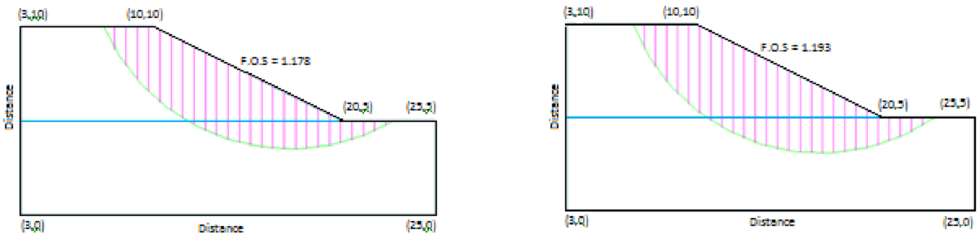
.(a) Morg–Price (b) Spencer

Figure 5.a, b dry soil in 2D slip surface with Morgenstern–Price and Spencer



.(a) Morg–Price (b) Spencer

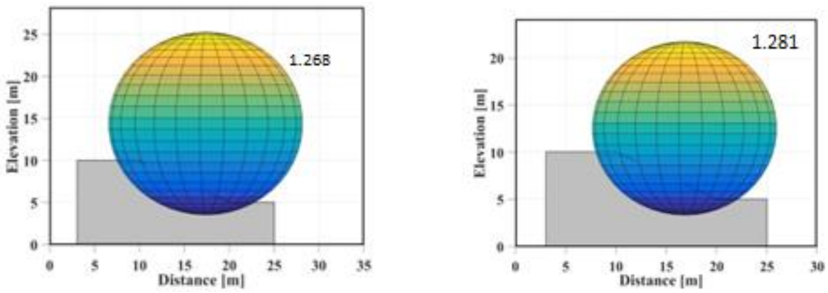
Figure 6.a, b soil in 2D with 1.0 m water depth below the surface with Morgenstern–Price and Spencer



.(a) Morg–Price (b) Spencer

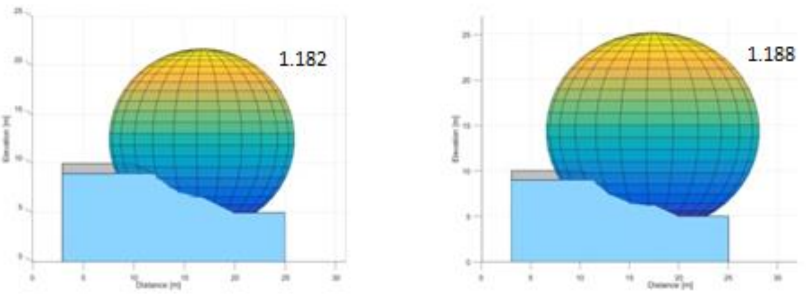
Figure 7.a, b water depth in the middle of the surface with Morgenstern–Price and Spencer

The flowing results for the above simple slope by using the gray wolf optimization method in both two and three dimensions slope stability was. Figure (8-a) and figure (8-b) presented the 3D critical slip surface in case of dry soil it found to be 1.268 and 1.281 by using Morgn–Price and Spencer methods respectively. The optimum slip surfaces with its factors of safety shows in both figure (9-a, b) when using the same slope with 1.0 m water level below the slope surface found to be 1.182 and 1.188 by using Morg–Price and Spencer methods respectively. When the water level be in slope middle the minimum FOS was found to be 1.240 in using the 3D method with Morg–Price method as shown in figure (10-a), and its value be 1.245 in Spencer method as shown in figure (10-b).



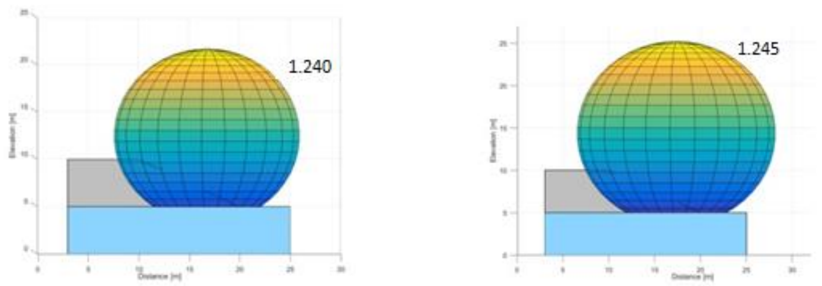
.(a) (b)

Figure 8.a, b 3D dry soil with Morgenstern–Price and Spencer critical slip surface



.(a) (b)

Figure 9.a,b 3D with 1.0 m water depth below the surface by using Morgenstern–Price and Spencer



.(a) (b)

Figure 10.a,b 3D slope with water depth in the middle of the surface using Morgenstern–Price &Spencer

7.2 Slopes results and discussions

The minimum factors of safety results were tabulated on table 5 for both two and three dimensions slip surface, which calculated by running the gray wolf optimization on MATLAB program by using four different slopes in three different situations. A comparison between the results for minimum factor of safety (FOS) in case 2D and 3D for the various configuration of the slope, it can seen that 3D slope stability results are more accurate to natural soil stabilized behavior.

The results showed that the three-dimensional safety factor is always greater than the two-dimensional safety factors with a variable percentage ranging from (4% to 10%). such as the simple slope with Morgenstern-price in the first row the 3D FOS is 1.268, while the 2D FOS is 1.198, indicating that the 3D is 5.84% percent better than the 2D.

In case of slope with benches when the water level be of middle of the slope the 2D factors of safety for both Morg–Price method's and Spencer were 1.186 and 1.216 respectively; and in 3D and Morg–Price method the value of F.O.S is equal to 1.252 this results is equal to the dry soil F.O.S values, because the water level is very low so the circular didn't cut the slope from the water level, this which happened also in the steep slope.

Table 5: results of factor of safety for all cases studied

No	Slope type	Method	Factor of safety					
			Two-dimension			Three-dimension		
			No water level	Middle height water level	1.0 m water level	No water level	Middle height water level	1.0 m water level
1	Simple	Morg-price	1.198	1.178	1.112	1.268	1.240	1.182
2		Spencer	1.205	1.193	1.129	1.281	1.245	1.188
3	Heterogeneous	Morg-price	1.322	1.293	1.189	1.429	1.396	1.324
4		Spencer	1.347	1.320	1.213	1.485	1.454	1.442
5	With benches	Morg-price	1.186	1.186	1.123	1.251	1.251	1.209
6		Spencer	1.216	1.216	1.155	1.291	1.271	1.256
7	Steep slope	Morg-price	1.054	1.054	0.960	1.096	1.096	1.021
8		Spencer	1.058	1.056	0.974	1.106	1.106	1.028

The ratio between the results in case of dry soil and the effect of the water in both 2D and 3D were found to be as follow:

- The ratio between the dry soil in 2D to dry soil in 3D was Ranges between maximum 9.3% as in the homogenous slope by using Spencer method and its minimum ratio was 4.38% as in the steep slope with Spencer method
- the ratio between 2D and 3D when the water depth was in the middle of the slope arrived to its maximum value in the homogenous slope using Spencer method it was 9.22%, But it achieved the lowest percentage in steep slope with Morgenstern-price method it was 2.05%
- When the water depth was1.0 m below the slope surface, the ratio between 2D and 3D reached its maximum value of 10.2% in the homogeneous slope using Morgenstern-price method, but it reached its lowest percentage of 4.97% in the simple slope using Spencer method

8. CONCLUSIONS

This paper demonstrates the prediction of the soil slip surface and its factor of safety by using optimization techniques. So the after simulating this problem with the Gray wolf behavior and running the program many times the following conclusion was obtained:

1) The MatLab software was developed and validated as a generalized three-dimensional stability analysis model.
2) the factor of safety was determined twice, once using Morg-price and once using Spencer, For four different types of slopes under three different conditions
3) in all presented slopes type The factors of safety calculated by the 3D slope stability presented results too close to the natural soil results
4) The GWO has higher computational efficiency, faster and easier than the other optimization techniques.

9. REFERENCES

1. Ali,N., Gad, S Abo-bakr, M (2021) "3-D Critical Slip Surface Based on Artificial Intelligent" geomate vol 21, pp, 99-106.
2. Bai, T., Hu, X., Gu, F. & Asce, A. M. (2019) "Practice of Searching a Noncircular Critical Slip Surface in a Slope with Soil Variability". 19, 1–13.
3. Donald, I.B., and Giam, P.S.K., (1995), The ACADS slope stability review, in Landslide, Sixth International Symposium on Landslides, Christchurch, p. 1665–1670.
4. Dorigo, M., and Stutzle, T. (2004), "Ant colony optimization", MIT Press,Cambridge, MA.
5. Fredlund, D. G., Krahn, J., & Pufahl, D. E. (1981), The Relationship between Limit Equilibrium Slope Stability Methods. Proceedings of the International Conference on Soil Mechanics and Foundation Engineering, pp. 409-416. Stockholm, Sweden
6. Gao, W. (2015), "Determination of the Noncircular Critical Slip Surface in Slope Stability Analysis by Meeting Ant Colony Optimization"
7. Huang, Y., Zhao, L., Xiong, M., Liu, C. & Lu, P. (2018) "Critical slip surface and landslide volume of a soil slope under random earthquake ground motions". Environ. Earth Sci. 77, 1–11.
8. Janbu, N. (1954), "Application of composite slip surfaces for stability analysis. In Proceedings of the European Conference on Stability of Earth Slopes", Stockholm, Sweden.
9. Janube N.(1973)," slope stability computations in Embankment-Dam Engineering" john Willy,P.47-86
10. Li, S. H., Wu, L. Z. & Luo, X. H. (2020), "Engineering Applications of Artificial Intelligence A novel method for locating the critical slip surface of a soil slope"94.
11. Li, Y. X. & Yang, X. L.(2019) "Soil-Slope Stability considering Effect of Soil-Strength Nonlinearity". 19, 1–10.
12. Mech. L. D., (1999) "Alpha status, dominance, and division of labor in wolf packs," Canadian Journal of Zoology, vol. 77, pp. 1196-1203.
13. Mirjalili S, Mirjalili SM, (2016) "Multi verse optimizer: a nature inspired algorithm for global optimization". Neural Comput Appl 27(2):495–513.

14. Mishra, M., Ramana, V. & Damodar, G.(2020), Teaching – learning-based optimisation algorithm and its application in capturing critical slip surface in slope stability analysis. Soft Comput. 24, 2969–2982.
15. Mishra, M. Multiverse (2020), "Optimisation Algorithm for Capturing the Critical Slip Surface in Slope Stability Analysis". Geotech. Geol.38, 459–474.
16. Morg, N., & Price, V. E. (1965), "The analysis of the stability of general slip surfaces". Geotechnique, London, 15(1), 79-93.
17. Muro. C, R. Escobedo, L. Spector, and R. Coppinger, (2011) "Wolf-pack (Canis lupus) hunting strategies emerge from simple rules in computational simulations," Behavioural processes, vol. 88, pp. 192-197
18. Nagendran, S. K., Ashraf, M., Ismail, M. & Tung, W. Y. (2019) "2D and 3D rock slope stability assessment using Limit Equilibrium Method incorporating photogrammetry technique"68, 133–139
19. Nash, D. (1987), "Comparative review of limit equilibrium methods of stability analysis. Slope Stability": Geotechnical Engineering and Geomorphology. John Wiley and Sons New York. p. 11-75.
20. Salah. A., Yannic .E, François .D, (2017) “MATLAB script for slope stability calculations with COMSOL Multiphysics”
21. Spencer,E.,(1967), "A method of analysis of the stability of embankments assuming parallel inter-slice forces". Geotechnique, 17, 11-26.
22. Yang, Y., Xing, H., Yang, X. & Zhou, J. (2016), "Determining the Critical Slip Surface of Three-Dimensional Soil Slopes from the Stress Fields Solved Using the Finite Element Method"
23. Yue Zhou, Shun-Chao Qi, Gang Fan. (2020) "water Topographic Effects on Three-Dimensional Slope". 1–24.