Al-Azhar University Civil Engineering Research Magazine (CERM)
Vol. (43) No. (1) January 2021

EFFECT OF THE PRESTRESSING FORCE ON THE FREQUENCY OF CRACKED AND UNCRACKED POST-TENSIONED CONCRETE SLABS

Fady Ibrahim Ezzat [a], Amr Abdelrahman [b], Ezzeldin Sayed-Ahmed [c]

[a] M.Sc. Candidate, Structural Engineering Department, Faculty of Engineering, Ain Shams University, Cairo, Egypt.
[b] Professor of Concrete Structures, Structural Engineering Department, Faculty of Engineering, Ain Shams University, Cairo, Egypt.
[c] Professor of Structural Engineering, Construction Engineering Department, School of Sciences and Engineering, The American University in Cairo, Cairo, Egypt.
[a] Corresponding Author E-mail: fadyibrahim@aucegypt.edu

تأثير قوة سبق الإجهاد على تردد البلاطات الخرسانية لاحقة الشد المشرخة و الغير مشرخة

الملخص العربى :
تتميز البلاطات الخرسانية لاحقة الشد بسماكة أقل من البلاطات الخرسانية المسلحة العادية مما يثير مشاكل الإهتزاز لمثل هذه البلاطات نظرا لتقليل جساءة القطاع. تقدم الأبحاث السابقة إستنتاجات مختلفة لموضوع تأثير قوة سبق الإجهاد على تردد العناصر الخرسانية لاحقة الشد، و يدرس هذا البحث سلوك البلاطات المشرخة و الغير مشرخة باستخدام برنامج العناصر المحددة ABAQUS. تم إستنتاج أن قوة سبق الإجهاد لا تؤثر على جساءة و تردد البلاطات الغير مشرخة نتيجة أحمال التشغيل، لكن تردد البلاطات الخرسانية المشرخة يتناسب طرديا مع قوة سبق الإجهاد لتقليل شروخ الشد بالعنصر الخرسانى و بالتالى زيادة جساءة و تردد البلاطة الخرسانية. يدرس أيضا هذا البحث طريقة الترخيم الاستاتيكى لحساب تردد البلاطات الخرسانية، وتم إستنتاج أن هذه الطريقة لا تقدم نتائج دقيقة.
الكلمات المفتاحية: البلاطات الخرسانية لاحقة الشد، تردد النمط الأول، الخرسانة سابقة الإجهاد، البلاطات المشرخة، البلاطات الغير مشرخة

ABSTRACT:

Post-tensioned concrete flat slabs are characterized with smaller thickness than ordinary reinforced concrete flat slabs, which could raise vibration problems for such slabs. However, the past research presented different findings for the influence of the prestress force on the frequency of prestressed concrete floors. This research investigates the effect of the prestress force on the frequency of cracked and uncracked prestressed concrete slabs using the finite element software ABAQUS. It is found that the prestressing force does not affect the frequency of uncracked prestressed concrete floors, but it increases the effective inertia of

cracked sections by closing such tensile cracks which increases the stiffness and the frequency of the section. In addition, the research studies the accuracy of the static deflection method in the prediction of the frequency of prestressed concrete slabs, and the method was found to give more conservative results than actual values.

Keywords: Post-tensioned concrete slabs, Frequency, Prestressed concrete, Cracked slabs, Uncracked slabs

INTRODUCTION

Prestressed concrete is used globally in several residential and commercial projects in different forms especially bridges and slabs for its advantages. Prestressed concrete structures offer some advantages over reinforced concrete structures such as less rebar reinforcement, uncracked section for service state, and smaller concrete thickness for slabs or large spans. The latest advantage provides slender concrete sections, which makes the vibration analysis essential to check the dynamic behavior of such slabs, as the smaller thickness and lower frequency makes slabs susceptible to resonance phenomenon where the frequency of the slab (f) is very close to the frequency of the dynamic force ($\bar{f}$), which may lead to very high dynamic displacement following (Equation 1), where (r) is the ratio between ($\bar{f}$) and (f).

1- $$Dynamic\ Displacement = \frac{Static\ Displacement}{1-r^2} \quad \text{(Equation 1)}$$

The first mode of motion of the slab is the most critical for its low frequency therefore, the frequency of the first mode of the slab shall not be less than the minimum allowable values stated by many references as [1] where Khan recommended the frequency of the slab not to be less than 8 Hz to avoid large dynamic displacement.

This research discusses the effect of the prestressing force on the frequency of cracked and uncracked post-tensioned concrete slabs. In addition, the paper studies the accuracy of the static deflection method in the estimation of the frequency of post-tensioned concrete slabs.

THE PRESTRESS LEVEL EFFECT ON FREQUENCY OF STRUCTURES

There is no final agreement reached in the past research for the effect of the prestress force on the frequency of prestressed concrete structures. It is confirmed by Kovalovs et al. [2] that there are three points of view which are:

THE PRESTRESSING FORCE AND BEAM'S FREQUENCY ARE DIRECTLY PROPORTIONAL TO EACH OTHER

Lu et al. [3] declared that the fundamental frequency of a 4m-long prestressed concrete beam increased only 0.43% after applying the prestressing force. Jang et al. [4] applied six prestress forces on an 8m-long cracked concrete beam, and it was found that the frequency of the cracked beam increased by increasing the prestress force to the point where the prestressing force could close all tensile cracks in the beam. When the beam turned uncracked, the prestress level did not influence the frequency of the beam. Noble et al. [5] confirmed the same conclusion of Jang et al [4] for the effect of the prestress force on the frequency of cracked prestressed concrete beams by applying various tendon eccentricities and prestress loads.

THE PRESTRESSING FORCE AND BEAM'S FREQUENCY ARE INVERSELY PROPORTIONAL TO EACH OTHER

Law et al. [6] simulated a prestressed concrete beam subjected to several prestress forces, and the authors confirmed that the frequency of the beam decreased with increasing the prestress force. However, the applied forces were too high which led to compression softening of concrete which decreased the elasticity modulus and the stiffness of the concrete section.

THERE IS NO RELATIONSHIP BETWEEN THE PRESTRESSING FORCE AND BEAM'S FREQUENCY

Noble et al. [7] applied dynamic tests on uncracked prestressed beams with the same properties as mentioned in his research [5]. The authors confirmed that the prestressing force did not affect the frequency of the beam. Goh et al. [8] used MATLAB to examine the parameters influencing the frequency of prestressed concrete beams. The authors applied five prestress forces on a 2.7m-long prestressed concrete beam confirming that there is no relationship between the prestressing force and the frequency of the beam. Bonopera et al. [9] investigated the effect of three different prestressing forces on the frequency of an uncracked prestressed concrete beam, and Bonopera confirmed that there are no tensile cracks or compression softening of concrete occurs in the beam under the applied prestress forces. Bonopera concluded that the prestressing force did not influence the frequency of the prestressed concrete beam significantly.

NUMERICAL MODELLING

ABAQUS is a finite element-based software used to model post-tensioned concrete slabs with different layouts and loads on the slab. Slabs are represented as solid elements of 8-node quadrilateral elements as shown in (Figure 1), and prestressing strands are represented as wire truss elements. Flat slabs are modelled in 3x3 bays as in (Figure 2), and their geometries are presented in (Table 1).

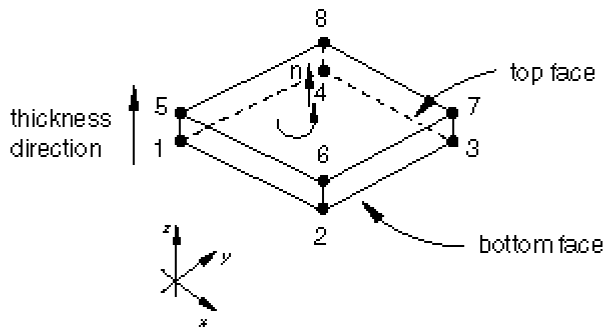

Figure 10 Nodes of Solid Elements [10]

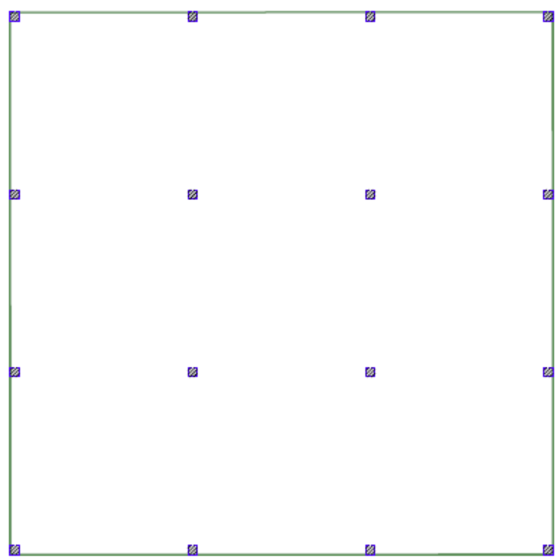

Figure 11 Layout of Modelled Prestressed Concrete Flat Slabs

Table 3 Properties of Modelled Prestressed Concrete Flat Slabs

Model number	Slab thickness (m)	Long span (m)	Short span (m)	Columns length (m)	Columns width (m)
Model 1	0.28	10	10	0.4	0.4
Model 2	0.225	7.2	7.2	0.45	0.45
Model 3	0.35	8	7.5	0.45	0.45
Model 4	0.3	9	9	0.6	0.6

Properties of concrete and prestressing steel are mentioned in (Table 2), and the stress-strain plot of concrete in tension and compression is shown in (Figure 3), and that for prestressing steel is shown in (Figure 4).

Table 4 Properties of Concrete and Prestressing Steel Used in Modelling

	Concrete	Prestressing steel
Density (kN/m^3)	23.544	76.518
Dynamic elasticity modulus (MPa)	34.79E+03	200E+3
Poison's ratio (υ)	0.2	0.0

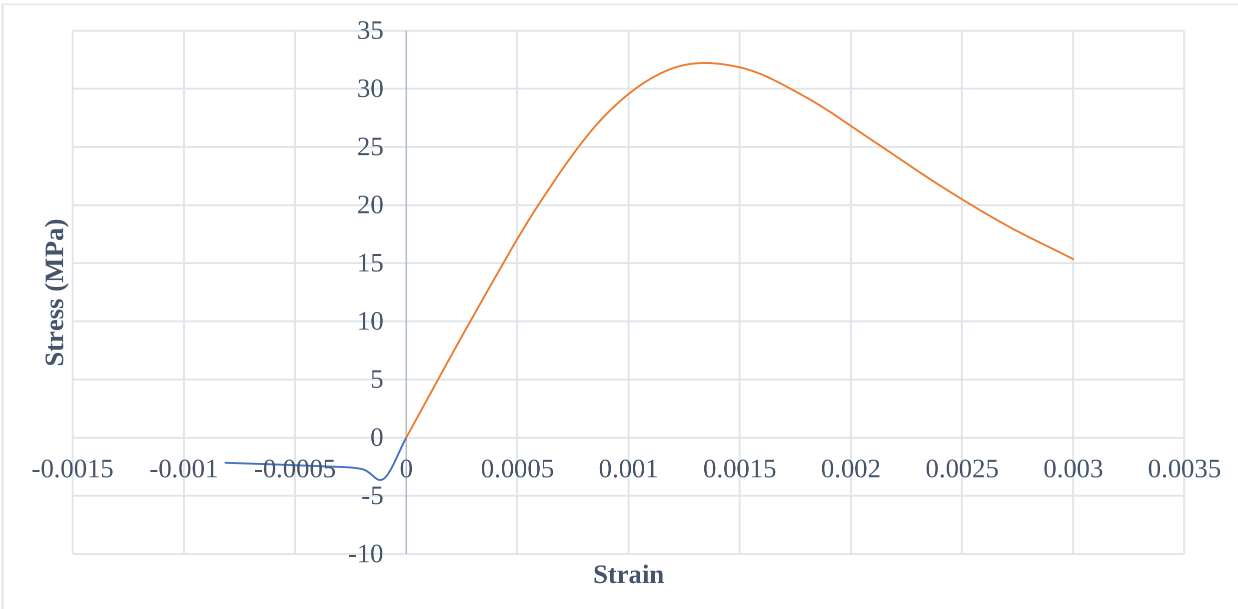

Figure 12 Stress-Strain Plot of Concrete

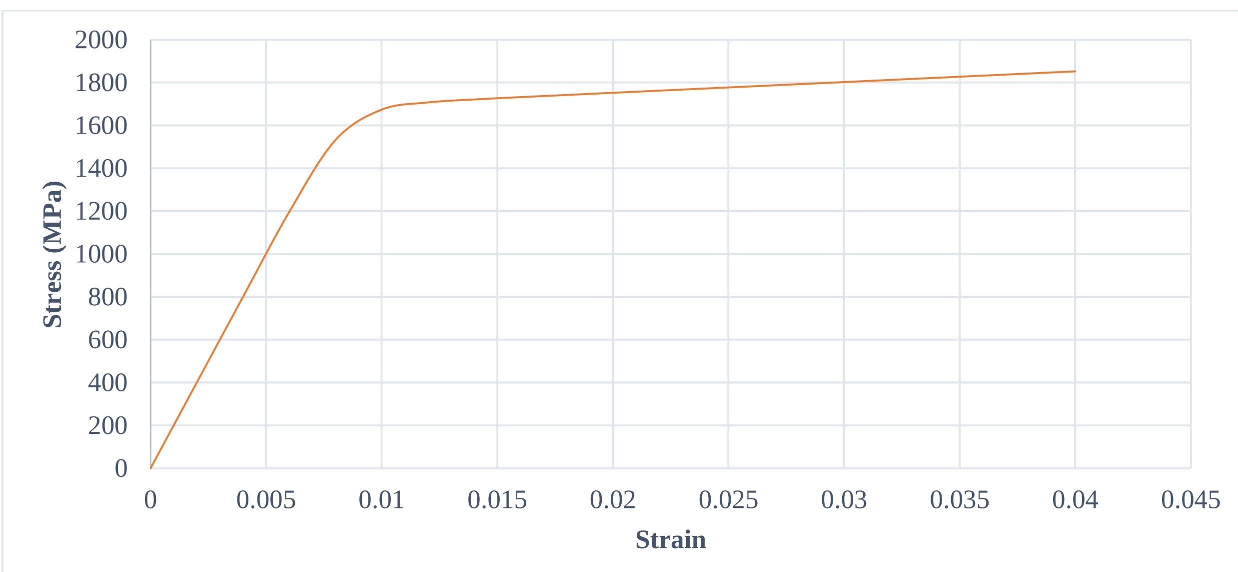

Figure 13 Stress-Strain Plot of Prestressing Steel

THE INFLUENCE OF THE PRESTRESSING FORCE ON THE FREQUENCY OF UNCRACKED POST-TENSIONED CONCRETE FLAT SLABS

Different prestressing forces are applied on modelled flat slabs to test the effect of the prestress force on the frequency of concrete slabs, and results are presented in (Table 3).

Table 5 Frequencies of Prestressed Slabs of Different Prestress Forces

Model number	High prestress force (Hz)		Average prestress force (Hz)		Zero prestress force (Hz)	
	Freq. (Hz)	Variation	Freq. (Hz)	Variation	Freq. (Hz)	Variation
Model 1	6.1898	0.38%	6.2137	0.0%	6.1266	1.42%
Model 2	10.729	1.04%	10.842	0.0%	10.805	0.34%
Model 3	12.916	0.84%	13.025	0.0%	13.029	0.03%
Model 4	9.5021	0.25%	9.5259	0.0%	9.4385	0.92%

Models of average prestress forces assure that slabs are uncracked and no compression softening of concrete occurs. The frequencies of slabs slightly decrease by applying high prestress forces due to compression softening effect, and frequencies slightly decrease when the prestressing forces are deactivated as slabs start a little to crack. However, it is confirmed that the prestress force effect is negligible as the highest variation shown in (Table 3) is 1.42%.

THE INFLUENCE OF THE PRESTRESSING FORCE ON THE FREQUENCY OF CRACKED POST-TENSIONED CONCRETE FLAT SLABS

Prestressed flat slab (Model 1) is loaded with higher loads to allow concrete tensile cracking, and results are shown in (Table 4).

Table 6 Frequencies of Cracked Prestressed Concrete Slabs

External load (kN/m^2)	Uncracked Slabs	Prestressed slabs		Non-prestressed	
	Frequency (Hz)	Frequency (Hz)	Variation	Frequency (Hz)	Variation
3	5.1866	5.0767	2.12%	4.8069	7.32%
4.5	4.82	4.6752	3.00%	4.2735	11.34%

Higher loads on slabs bring higher straining actions and bigger tensile cracks, and prestressing strands work on closing such cracks which increases the effective inertia of the concrete section. Results in (Table 4) show the major effect of the prestress forces on the frequency of cracked prestressed concrete slabs.

THE ACCURACY OF THE STATIC DEFLECTION METHOD IN DETERMINING THE FREQUENCY OF CONCRETE SLABS

The static deflection method (SDM) is mentioned in PCI [11] and SCI [12] following (Equation 2), and it is stated to determine the first mode's frequency (f) of concrete slabs.

$$f(\mathrm{Hz}) = 0.18\sqrt{\frac{g}{\Delta}} \qquad \text{(Equation 2)}$$

where (g) is the gravitational acceleration 9.81 m/s^2, and (Δ) is the deflection in meters due to service gravity loads without the prestress effect.

This method is investigated for (Model 1) under different loads to examine its accuracy in predicting the frequency of concrete slabs, and results are shown in (Table 5).

Table 7 Examining the Static Deflection Method to Determine the Frequency of Concrete Slabs

External load (kN/m^2)		0	3	4.5
Uncracked slabs	FEM freq. (Hz)	6.2653	5.1866	4.82
	Deflection (mm)	8.295	15.82	19.59
	SDM freq. (Hz)	6.1901	4.4823	4.0280
	Variation %	1.20%	13.58%	16.43%
Cracked slabs	FEM freq. (Hz)	6.1266	4.8069	4.2735
	Deflection (mm)	10.16	29.16	51.35
	SDM freq. (Hz)	5.5932	3.3015	2.4879
	Variation %	8.71%	31.32%	41.78%

The static deflection method is shown to present inaccurate results following (Table 5) which shows the error percentage may reach 41%.

CONCLUSION

This paper concludes the following:

1- Prestress forces shall not be too high to avoid compression softening of concrete.
2- Prestress forces increase the frequency of cracked concrete slabs.
3- Prestress forces do not affect the frequency of uncracked concrete slabs.
4- The static deflection method may present inaccurate results for the frequency of concrete slabs

REFERENCES

[1] A. Khan, "Frequency Estimation of Pre-stressed and Composite Floors," publications@city.ac.uk, London, UK, 1996.

[2] A. Kovalovs, S. Rucevskis, P. Akishin and J. Kolupajevs, "Numerical Investigation on Detection of Prestress Losses in a Prestressed Concrete Slab by Modal Analysis," *IOP Conference Series: Materials Science and Engineering,* vol. 251, no. 1, 2017.

[3] Z. Lu and S. L. , "Identification of Prestress Force From Measured Structural Responses," *Mechanical Systems and Signal Processing,* vol. 20, no. 8, pp. 2186-2199, 2006.

[4] J. B. Jang, H. P. Lee, K. M. Hwang and Y. C. Song, "A Sensitivity Analysis of The Key Parameters for The Prediction of The Prestress Force on Bonded Forces," *Nuclear Engineering and Technology,* vol. 42, no. 3, pp. 319-328, 2010.

[5] D. Noble, M. Nogal, A. O'Connor and V. Pakrashi, "The Effect of Prestress Force Magnitude and Eccentricity on the Natural Bending Frequencies of Cracked Prestressed Concrete Beams," *Journal of Physics,* vol. 628, no. 1, 2015.

[6] S. Law and Z. Lu, "Time Domain Responses of a Prestressed Beam and Prestress Identification," *Journal of Sound and Vibration,* vol. 288, no. 4-5, pp. 1011-1025, 2005.

[7] D. Noble , M. Nogal, A. O'Connor and V. Pakrashi, "The Effect of Prestress Force Magnitude and Eccentricity on the Natural Bending Frequencies of Uncracked Prestressed Concrete Beams," *Journal of Sound and Vibration,* vol. 365, pp. 22-44, 2016.

[8] L. Goh, A. Rahman, N. Bakhary and B. Ahmad, "Factors Influencing Natural Frequencies in a Prestressed Concrete Panel for Damage Detection," *Jurnal Teknologi,* vol. 69, no. 3, pp. 51-57, 2014.

[9] M. Bonopera, K. Chang, C. Chen, Y. Sung and N. Tullini, "Experimental Study on the Fundamental Frequency of Prestressed Concrete Bridge Beams with Parabolic Unbonded Tendons," *Journal of Sound and Vibration,* vol. 455, no. 200, pp. 150-160, 2019.

[10] "https://abaqus-docs.mit.edu/," [Online].

[11] PCI Design Handbook, 7th Edition ed., Chicago: Precast/Prestressed Concrete Institute, 2010.

[12] SCI Publication P354, Design of Floors for Vibration: A New Approach, Berkshire: The Steel Construction Institute, 2009.