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THE ANALYSIS OF SHELL STRUCTURES UNDER CONSIDERATION OF LIQUID-STRUCTURE INTERACTION AND THE HYGROTHERMAL BEHAVIOUR OF CONCRETE

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ملخص البحث:
بالإضافة الي الأحمال الميكانيكية و احمال التشغيل تتعرض المنشآت الخرسانية الي الأحمال البيئة المحيطة مثل الحرارة و الرطوبة. بفحص الأضرار في المنشآت المنفذة غالبا نخلص الي نتيجة عامة، انه بالإضافة الي الأحمال الديناميكية مثل الزلازل و الرياح أيضا الرطوبة والحرارة تتسبب في توليد اجهادات تكون هي بداية نشاة الأضرار في المنشأ . أكثر من ذلك اعتبار التأثير الديناميكي المتبادل بين المنشأ و السائل المختزن في الخزانات اثناء الزلازل مهم جدا للحصول علي معلومات اكثر عن الأضرار المتولدة اثناء الزلازل في منشآت تخزين السوائل.

ABSTRACT
Reinforced concrete structures are subjected to environmental and operational loads in addition to mechanical loads during their service life. Investigations of damages on existing structures often conclude that in addition to dynamic loads from wind and earthquake also moisture and thermal induced stresses have initiated the damages. Furthermore, the considering of dynamic liquid-structure interaction during an earthquake excitation is very important for getting a closer-look on the damages in liquid storage tanks. In the present study a concept to model the thermal and hygric transport across concrete shell walls has been developed, and the resulting structural behavior has been analyzed and compared with experimental results. Furthermore, the interaction of liquid and structure has been modeled by a finite element formulation, which allows simulating the time variation of liquid pressure on the tank walls during an earthquake or any kind of dynamic excitation. The analyzed results are in a good agreement with experimental and analytical results.

Keywords: moisture and temperature in concrete, liquid-structure interaction

1. INTRODUCTION
Undoubtedly the cracking of concrete is one of the main reasons for the limitation of life-time of reinforced concrete structures. As well the serviceability may be no longer guaranteed, if storage containments have lost their liquid-tightness. Moreover, the structural eigenfrequencies may be shifted closer to any kind of dynamic excitation, thus reducing the ultimate limit stability in case of resonant action. In general, a coincidence

of dynamic action due to wind and life loads together with moisture and temperature induced stresses has to be looked upon as the initiator of a damaging process. This especially holds true for tank structures, which may be stressed by liquid filling combined with thermal and hygric effects, and which in earthquake-active regions additionally may be affected by dynamic excitation. For the numerical simulation of such damaging processes in reinforced concrete shell structures, the present coupled thermo-hygro-mechanical material model has been discussed in detail in Wörmann R. (2004) using a four-noded, isoparametric, Reissner-Mindlin-type finite shell element, based on a middle-surface orientated shell theory with rotational variables developed by Montag, U. (1996), as basis for the implementation. For the realistic recording of the physically non-linear reinforced concrete behaviour the multi-layered concept is employed in thickness direction of the shell element. The material model of implemented shell element is according to Beem H. et al (1996) and Harte, R et al (2003) in the finite element system FEMAS 2000.

For modeling of liquid, two finite element formulations are possible: Lagrangian formulation of motion (i.e. particles are followed in their movement) and Eulerian formulation is usually used for a fluid flow analysis (behavior of the fluid at a particular position in space is of interest). Both formulations discussed in detail by Chung, T. J. (1978) and Zienkiewicz, O. C. et al (1991).

The aim of present study is to analyse the concrete shell structure under the effect of heat and moisture, also considering fluid-structure interaction under dynamic or earthquake excitation.

2. HYGROTHERMAL CONCRETE MODELLING

In addition to mechanical loads, porous media like concrete are as well exposed to thermal and hygric influence caused by production methods, operational conditions or atmospheric nature. The resulting thermal and hygric unbalance within a concrete body or between the body and its environment leads to transient processes. These are characterized by a combination of different transport und storage mechanisms.

Two coupled equations of balance can be formulated to determine the temperature and moisture distribution, which vary over time and location in the concrete. An energy balance is defined for the heat and a mass balance for the moisture situation. Thereby vapor diffusion and liquid transport are taken into account separately. The temperature ϑ and the relative humidity h are employed as driving potentials. Starting from the separately evaluated equations of balance and considering the relevant interactions between heat and moisture the following coupled system of non-linear differential equations is received:

$$
\begin{aligned}
(\rho_c c_c + w c_w)\,\dot{\vartheta} = \operatorname{div}\left[\left(\boldsymbol{\lambda}_t + r\boldsymbol{\delta}_p \frac{\partial \mathrm{p}_{sat}}{\partial} h\right)\cdot \operatorname{grad}\vartheta \right] \\
+ \operatorname{div}\left[r\boldsymbol{\delta}_p p_{sat} \cdot \operatorname{grad}\ h\right],
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
\frac{\partial w}{\partial h}\dot{h} = \operatorname{div}\left[\boldsymbol{\delta}_p \frac{\partial p_{sat}}{\partial} h \cdot \operatorname{grad}\ \vartheta\right] \\
+ \operatorname{div}\left[\left(\boldsymbol{\delta}_p\, p_{sat} + \mathbf{D}_w \frac{\partial w}{\partial h}\right) \cdot \operatorname{grad}\ h\right],
\end{aligned}
\tag{2}
$$

where ρ_c denotes the density of dry concrete, c_c the specific heat capacity of the aggregate matrix, w the water content, c_w the heat capacity of water, $\boldsymbol{\lambda}_t$ the thermal conductivity matrix, r the specific heat of evaporation, $\boldsymbol{\delta}_p$ the vapor permeability matrix of concrete, p_{sat} the saturation vapor pressure and $\mathbf{D}_w$ the liquid diffusivity matrix. For modelling the material parameters in equations (1) and (2) temperature- and moisture-dependent formulations have to be employed.

The differential equations (1) and (2) together with the initial and boundary conditions form the initial boundary value problem. For numerical solution this has to be consistently linearized and converted into a finite-element-formulation. A product assessment in room and time domain is applied for the discretisation. Lagrange interpolation polynomials are used for the spatial discretisation and for the time integration the collocation method is employed. The procedure in time corresponds to the well-known-scheme. Result of the transfer is the linearized, discrete form of the equations of balance, giving the iterative increments of the nodal state variables in each time step as follows:

$$
\begin{bmatrix} \mathbf{K}_{tt}^{\Omega} + \mathbf{K}_{tt}^{\Gamma} & \mathbf{K}_{th}^{\Omega} + \mathbf{K}_{th}^{\Gamma} \\ \mathbf{K}_{ht}^{\Omega} + \mathbf{K}_{ht}^{\Gamma} & \mathbf{K}_{hh}^{\Omega} + \mathbf{K}_{hh}^{\Gamma} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\vartheta}_e^{i+1} \\ \Delta \mathbf{h}_e^{i+1} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_t \\ \mathbf{F}_h \end{bmatrix} \tag{3}
$$

Herefrom a calculation method has been developed and implemented in the new program module HygTherm to simulate the balance processes in shell structures, which mostly occur in thickness direction. Different model versions have been integrated, which are distinguished by the way of the formulation of the material parameters.

3. TIME VARIANT MODELING OF REINFORCED CONCRETE SHELL STRUCTURES

For the numerical simulation of such damaging processes in reinforced concrete shell structures, first a coupled hygro-thermal-mechanical material formulation has been developed. The basis is a time-invariant model for reinforced concrete subjected to short-term loads, including the essential physical non-linear features. Coupled balance equations are formulated to determine the temperature and moisture distribution, which vary over time and location in the shell's thickness direction, and which are considered within a time-variant multi-level-iteration-strategy. The resulting state variables are being used to determine the thermal and moisture induced strains and resulting stresses on the material point level, under the assumption of the decomposition of the total strains. This allows considering the existing eigenstresses in addition to the constraint effects in the modeling of such procedure.

A four-noded, isoparametric, Reissner-Mindlin-type finite shell element, based on a middle-surface orientated shell theory with rotational variables developed by Montag, U. (1996), is used as basis for the implementation. For the realistic recording of the physically non-linear reinforced concrete behaviour the multi-layered concept is employed in thickness direction of the shell element. Hence the shell is subdivided into a finite number of layers, where the components concrete and reinforcing bars are characterized by separate layers (Figure 1).

The hygro-thermal states in the cross section are determined by a corresponding modelling in thickness direction of the shell. Therefore, the program module HygTherm was integrated into the shell element via an interface (Figure 2). Thus, on the integration point level the calculated state variables ϑ and h in the hygrothermal layers must be transformed to the concrete and steel layers. This conversion becomes necessary because of different possible locations of reinforcement, of a deviating number of layers in the different calculation sequences and of a not regular distribution of the layer thickness.

The resultant non-linear strain distributions in the respective concrete cross sections and the corresponding reinforcement layers are determined from the transient temperature and moisture distributions. The free strains are computed both for the thermal and the hygric case of the product of the respective coefficients of expansion with the rate of the state variables. These read in the unidimensional case:

$$\dot{\varepsilon}^{t} = \alpha_t \cdot \dot{\vartheta} \tag{4a}$$

and

$$\dot{\varepsilon}^{sh} = \alpha_{sh} \cdot \dot{\mathrm{h}} \tag{4b}$$

In these rate formulations α_t denotes the thermal coefficient and α_{sh} the shrinkage coefficient. Under the assumption of an additive decomposition of the total strains, the load-independent strains, which are distributed non-linear over the cross section, are integrated in a time-variant multi-level-iteration-strategy on the material point level. The strategy, based on the layered concept, is used for the construction of the matrices and vectors within an incremental-iterative solution technique for the calculation of geometrical and physical non-linear load-displacement-paths. In the constitutive laws the essential physical non-linear features are included, as tension cracking, concrete compression behaviour, elasto-plastic reinforcement behaviour and bond effect Noh, S. (2002). The distribution of load independent strains in the cross section are considered realistically by the presented procedure.

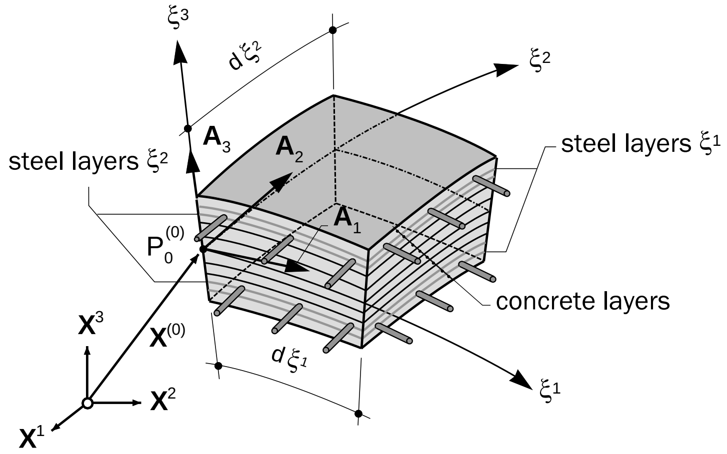

Figure 1: Structural reinforced concrete model in the finite shell element

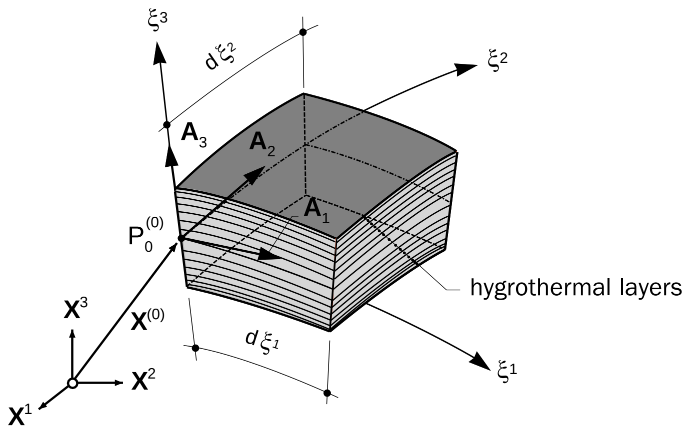

Figure 2: Hygrothermal concrete model in the finite shell element

4. MODELING OF LIQUID SLOSHING

For getting a closer-look on the damages caused by dynamic liquid-structure interaction by liquid storage tanks during an earthquake excitation, a finite element model of liquid medium has been developed. The concept of analysis by such structures is the subdivision of the entire system to two sub-regions: the structure region and the liquid region, and applying a numerical model such as finite element formulation. Then a one-step calculation will be carried out to solve the entire system, namely liquid and shell.

For modeling of liquid, two finite element formulations are possible: displacement formulation and pressure formulation, in other word Lagrangian formulation of motion (i.e. particles are followed in their movement) and Eulerian formulation is usually used for a fluid flow analysis (behavior of the fluid at a particular position in space is of interest) Alexander M. (2014). Another problem arises due to the Eulerian description of the solid equations, which brings along convective terms. In particular close to the interface, However Stefan et all (2017) derived numerical schemes of second order in space and time for interface problems with moving interfaces.

By displacement formulation “Lagrangian formulation”, the liquid will be considered as an elastic body without shear strength. In this case, it is not required to model the structure-liquid interface coupling in comparison with the pressure formulation “Eulerian formulation”, and the modal analysis can be carried out easier. The pressure formulation is more suitable for non-linear structural analysis, but it is more complicated by calculation procedure, because of the asymmetric stiffness matrix of the coupled system.

However, for displacement formulation “Lagrangian formulation”, The equilibrium relation between the own weight b_i , the stress σ_{ij} and the velocity $\dot{u}_i$ of an infinitesimal volumetric liquid element in *i*-direction gives the following dynamic equilibrium equation:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + b_i = \rho \frac{d\dot{u}_i}{dt} \tag{5}$$

With

$$\frac{d\dot{u}_i}{dt} = \frac{\partial \dot{u}_i}{\partial t} + \frac{\partial \dot{u}_i}{\partial x_j} \dot{u}_j \tag{6}$$

For a liquid storage tank the convective part $\frac{\partial \dot{u}_i}{\partial x_j} \dot{u}_j$ of Eq. (6) can be neglected Zienkiewicz, O. C. et al (1991).

For an incompressible liquid the stress tensor σ_{ij} will be expressed as follows;

$$\sigma_{ij} = -p\ \delta_{ij} + \lambda\ d_{kk}\ \delta_{ij} + 2\mu\ d_{ij} \tag{7}$$

Chung, T. J. (1978). Where p is the liquid pressure, d_{kk} and d_{ij} are the rate of deformation tensor and δ_{ij} is Cronecker-Delta. λ and μ are the viscosity Stok’s constants, which have the following relation

$$3\lambda + 2\mu = 0 \tag{8}$$

For a shear less liquid according to Eq. (7) the following relation is valid

$$\frac{\partial \sigma_{ij}}{\partial x_j} = -\frac{\partial p}{\partial x_j} \delta_{ij} \tag{9}$$

Under isothermal conditions as well as an assumption of a little compressibility of liquid the relation in Lagrange-coordinates between the liquid pressure p and the volumetric strain (dilatation e) is given as:

$$p = K \cdot e = K(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) \qquad (10)$$

Zienkiewicz, O. C. et al (1991), where K is the compression modulus and $\varepsilon_{ii} = \dfrac{\partial u_i}{\partial x_i}$ is the strain, Through the substitution from Eq. (9) and Eq. (10) in Eq. (5) the dynamic equilibrium equation in *i*-direction will take the following form:

$$K \frac{\partial^2 u_i}{\partial^2 x_i} + \rho \frac{d\dot{u}_i}{dt} - b_i = 0 \qquad (11)$$

The finite element formulation of this equation by means of Galerkin-procedure Chung, T. J. (1978) becomes

$$-\int_\Omega \nabla \mathbf{\Phi}^T K \, \nabla \mathbf{\Phi} \, d\Omega \, \mathbf{v} - \int_\Omega \mathbf{\Phi}^T \rho \, \mathbf{\Phi} \, d\Omega \, \ddot{\mathbf{v}} + \int_\Omega \mathbf{\Phi}^T \mathbf{b} \;\; d\Omega = 0 \qquad (12)$$

or

$$\mathbf{K} \; \mathbf{v} + \mathbf{M} \; \mathbf{v} = \mathbf{F} \qquad (13)$$

with

$$\mathbf{K} = \int_\Omega \nabla \mathbf{\Phi}^T K \; \nabla \mathbf{\Phi} \; d\Omega \;\; , \qquad (14)$$

$$\boldsymbol{M} = \int_\Omega \boldsymbol{\Phi}^T \rho \, \boldsymbol{\Phi} \, d\Omega \;\; , \qquad (15)$$

$$\mathbf{F} = \int_\Omega \mathbf{\Phi}^T \mathbf{b} \;\; d\Omega \qquad (16)$$

$$\nabla = grad\left(\cdots\right), \qquad (17)$$

and $\mathbf{\Phi}$ the shape function matrix of eight nodes volumetric element.

Contrarily to Housener Formals Meskouris, K. (1999), in which the eigenfrequencies of liquid are more dependent on the tank radius than the fill height, the liquid element developed here -with the assumption of liquid as an elastic body without shear strength - depends on the fill height more than the tank radius. In this case the consideration of shear modulus k_{rot} according to

$$\sigma_{ij} = k_{rot}\,\Omega_{ij} \tag{18}$$

with

$$\Omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) \tag{19}$$

and k_{rot} = 0.25 k, where k is the compression modulus, for calibration of results is very useful Brüggemann M. (2002).

By means of the presented formulation, the time dependent distribution of liquid pressure during any dynamic excitation can be simulated.

5. NUMERICAL VERIFICATION

5.1. HYGRIC BEHAVIOUR

The testing of the accuracy of the developed model was primarily performed by comparing experimental results with the results from numerical simulations. The different specific components of the model have been verified separately. Further examples are given in Wörmann, R. (2004).

To analyze the influence of moisture gradients on the behaviour of concrete structures, a one-sided concrete cantilever has been subjected to a non-uniform moisture environment situation at the Institute for Material Sciences at the Technical University Munich Springenschmid, R. Et al (1997). Within the series of experiments concrete with different mixtures have been examined. Figure 3 shows the graphics of the concrete cantilever. The beam was admitted with water on the bottom while on the top face it was subjected to constant relative air humidity of 65 %. The lateral surface of the beams is isolated from moisture and heat transmission, so that only a unidimensional humidity transport could have been possible.

While the water adsorption results in a swelling on the bottom, the water delivery leads to a shrinkage on the top face. The resultant change of curvature causes a raising of the cantilever at the free end. This raising occurs, though the dead weight will oppose the plate raising (German: “Aufschüsseln”). The measured development of raising over time could be confirmed by the numerical simulation (Figure 4).

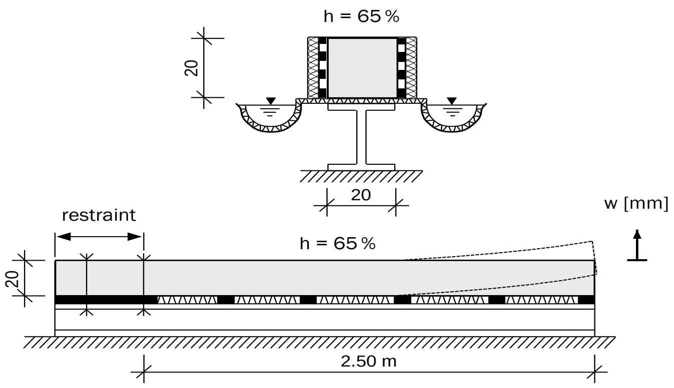

Figure 3: Concrete cantilever subject to hygric gradient: System by Springenschmid, R. Et al (1997)

A calibration of the numerical results to the experimental data is possible by the approach of modified entry parameters, for example of the moisture coefficient of dilatation. These phenomena have been often observed in recent years at reinforced concrete lane ceilings on highways. The resulting damages can be explained by the hygric gradient and precaution means then will be possible in future.

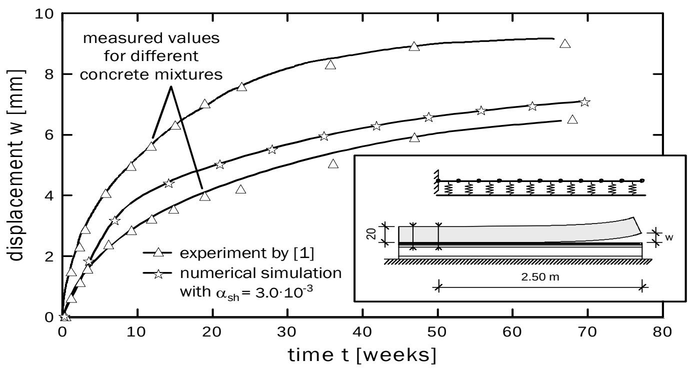

Figure 4: Concrete cantilever subject to hygric gradient: Raising of the free end.

5.2. HYGRO-THERMAL ACTION ON A TANK WALL

To verify the practicability of the developed software tools a realistic tank structure has been analysed. It has been assumed that hydrostatic liquid pressure and pre-stressing will completely balance each other. Thus, both load cases could be neglected, and besides the hygric action only dead-weight had to be considered. Figure 5 shows the overall dimensions and the concrete section with reinforcement bars.

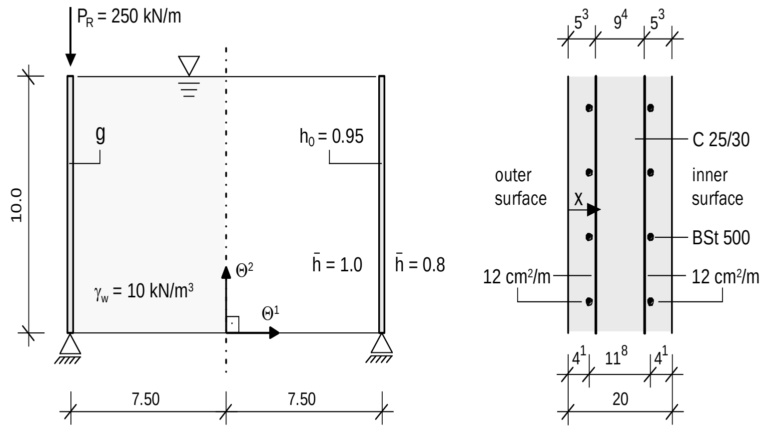

Figure 5:

The hygric boundary conditions auf h = 1.0 on the inner surface (water fill) and h = 0.8 on the outer surface (atmosphere). The hygric expansion coefficient has been taken to $\alpha_{sh} = 2.0 \cdot 10^{-3}$). Figure 6 shows the instationary distribution of the resulting nonlinear hygric strains across the shell thickness, and Figure 7 the resulting crack pattern of the different layers.

The look at the displacement pattern shows the typical behavior of cylindrical shells under thermal action with the well-known edge bending (Figure 8). The hygric action yields the same phenomenon, with nearly homogeneous bending moments in circumferential and meridional direction across the whole shell, which fade away close the edges. This is accompanied by further edge effects, like large circumferential forces, forcing the shell to crack. The Figure as well shows the long-term evaluation of displacements.

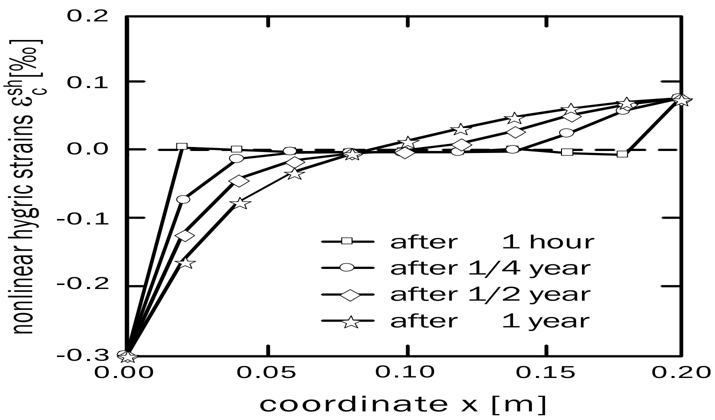

Figure 6:

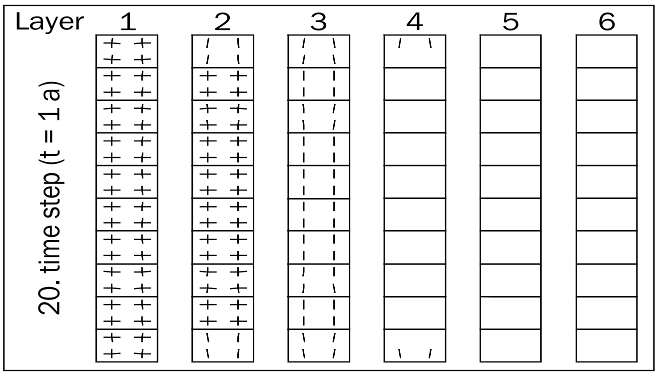

Figure 7:

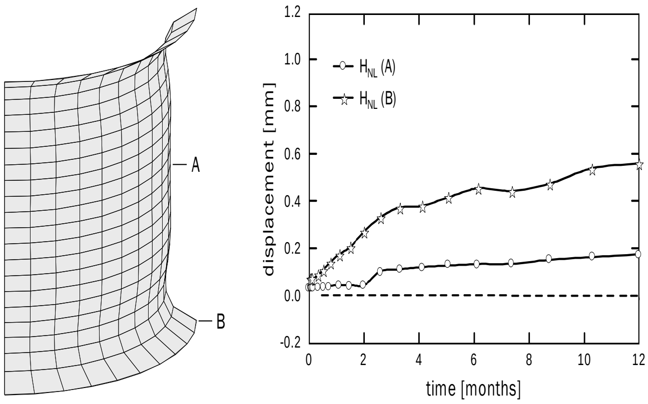

Figure 8:

5.3. LIQUID SLOSHING

For verifying of liquid element in static analysis, a rigid wall storage tank with a water fill ($K=22.5 \cdot 10^5$ kN/m^2), 5m radius and 9m height has been considered. The discretisation of water occurred with eight elements in radial direction, sixteen elements in circumferential direction and nine elements along the height. The water pressure in the middle point of elements along the height are presented in Figure (9) as a result of the liquid element discretisation. As expected, the results are coincident with the hydrostatic pressure.

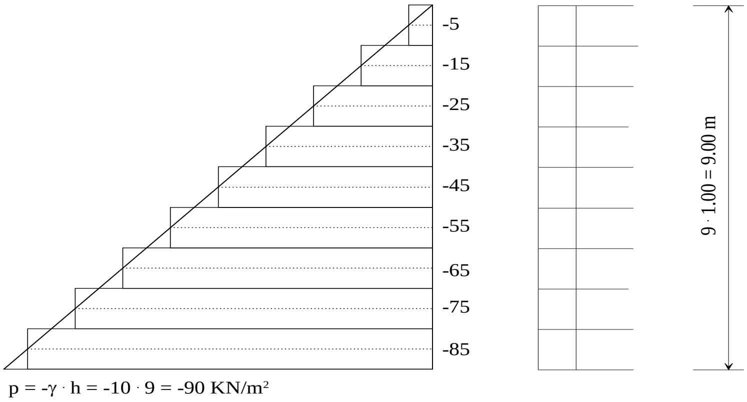

Figure 9: The water pressure under the own weight

For studying the dynamic behavior of the liquid element, the eigenfrequency of tank fill has been calculated. The analysis considered two values for the rotation factor K_{rot} (5.5 and 0.0). Figure (10) shows the static deformation and the first eigenform of tank fill. The angular frequency ω=1.9 rad/sec. Is coincident with the result applying Housner's formula Meskouris, K. (1999).

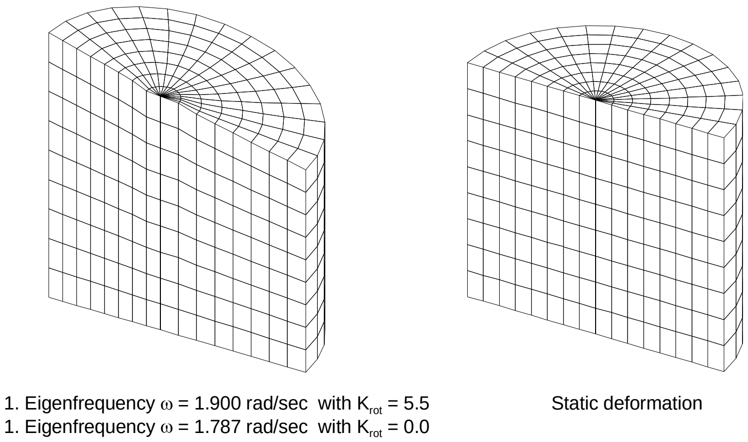

Figure 10: First Eigenform and static deformation of liquid

6. CONCLUSION

- For developing a multi-layered in thickness direction hydrothermal finite element, the three-dimensional field temperature and moisture values of continuum are transformed in two-dimensional field values of shell structures, this is fulfilled through replacing the third dimension by multi-layered in thickness direction shell element concept.
- So, the developed hydrothermal finite element works as interface element for calculating the nodal values of temperature and moisture, which varied in time and location within shell thickness, these calculated nodal values are used as case of loading acting on the concrete shell structure. Through coupling these nodal values with mechanical multi-layered in thickness direction finite shell element, then the corresponding straining actions will be produced.
- Thus, the numerical investigations of expected or existed concrete damages

deduced by humidity and temperature in reinforced concrete shell structures can be identified.

- Moreover, by the developed finite liquid element, a one-step calculation can be carried out to solve the entire system, namely liquid and shell during any dynamic excitation.
- However, the numerical applications using the present models are in a god agreement comparing with the experimental and analytical results.

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