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EFFECT OF CIRCULAR WEIR HEIGHTS AND DIAMETERS ON THE DISCHARGE COEFFEICIENT AND EFFICIENCY OF TAIL ESCAPE STRUCTURE

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ملخص

تعتبر المهربات احدى المنشأت المائيه الهامه التى يتم تنفيذها فى نهايه الترع والمصارف على شبكة الرى والصرف المصريه للتخلص من التصرفات وارتفاع المناسيب الزائده وقت أقل الاستخدامات للحفاظ على جسور الترع من الانهيار . وتتكون المهربات فى معظم الاحيان من هدار دائرى مع فتحه أسفله فى الامام وماسوره أخرى تعمل كبربخ فى الخلف ويهدف هذا البحث إلى دراسه تأثير التغير فى قطر وارتفاع الهدار الدائرى الرأسى على معامل التصرفات ولأجراء هذا البحث فقد تم تنفيذه بمعمل الهيدوليكا بكليه الهندسه جامعه الزقازيق على هدار أسطوانى بأقطار 10,16سم،12,70سم، 15,24 سم بأرتفاعات 29سم، 31,50سم، 34سم مع وجود فتحه دائريه فى قاع الهدار فى الأمام بقطر 2,60 سم وماسوره فى الخلف لتصريف التصرفات تعمل كبربخ بقطر 7,60سم وقد تم اجراء أكثر من 250 تجربه بالمعمل عند تصرفات وارتفاعات للمياه مختلفه وكانت أهم النتائج هى :-

1-معامل التصرفات للهدار الرأسى الأسطوانى يزداد مع زياده ارتفاع وقطر الهدار.

2-فى حاله تشغيل الهدار الأسطوانى الرأسى مع الفتحه الاماميه بالقاع فأن متوسط معامل التصرفات هو 0,6 عندما يكون ارتفاع المياه أعلى عتبة الهدار الى قطر الماسوره هو 0,10 وارتفاع المياه امام الفتحه السفليه الى قطرها يساوى 0,10. وقد اثبت البحث ان زيادة نسبة أرتفاع المياه اعلى عتب الهدار الى ارتفاع الهدار عن 0.10 تقلل من كفاءة الهدار الأسطوانى الرأسى والعكس صحيح.

3-فى حاله تشغيل الهدار مع فتحه الماسوره الاماميه بالقاع فأن تصرف الهدار يصل إلى 72% من التصرف الكلى.

4-تم تطوير معادلات رياضيه تستخدم لحساب معامل التصرف فى حاله تشغيل الهدارمنفردا أو فى حاله تشغيل الهدار والماسوره الأماميه معا.

ABSTRACT

At the end of canals and drains constructed tail or end escapes to remove the excess discharge. Its constructed from vertical circular weir, upstream pipe hole and pipe culvert connected at downstream bottom of circular weir. The main objectives of this research are studying effect of circular weir heights, and diameters on the discharge coefficients of circular weir, upstream pipe hole, and pipe culvert at different flow parameters. The research conducted through experimental, dimensional analysis and statistical analysis. The diameters of circular weir are 10.16cm, 12.70cm and 15.24cm with different heights of circular weir 29cm, 31.5cm , and 34.0cm. Pipe hole have diameter 2.6cm fixed at upstream bottom of weir and pipe culvert have diameter 7.6cm in downstream. The research results indicated that the increases in heights and decreases in weir diameters causes increases in discharge coefficient. For combined weir, pipe hole the optimum ratio of (Hw/P), and (ho/do) are 0.1, 10 respectively which give average discharge coefficient equal 0.6. The increase (Hw/p) more than 0.1 decreases from efficiency of weir and vise verse for upstream pipe hole.

INTRODUCTION

In Egypt, the end irrigation structures control for head and discharge at the end of irrigation and drainage networks. Also help to protect earthen water way in upstream against failure due to remove excess discharge to downstream. The present research study combined of upstream pipe hole, vertical circular weir and pipe culvert. Different previous research studies circular weir, and pipe culvert individual but we combined between the three hydraulics tools to deduce optimum design to hydraulic characteristics parameters. Chanson, H. and Montes, J.S.[1]described new experiments of circular weir outflows with eight cylinder sizes for several weir heights and for five types flow conditions. They decided the circular weir affect on the flow characteristics in upstream weir. Rahim, N. O. A., et al.[2] improved that experimentally the discharge coefficient for circular weir was greater than semi- circular weirs under free flow and submerged flow conditions. Ghobadian, R. and Meratifashi, E.[3] modified theoretical stage-discharge relation for circular sharp crested weirs. Gamargo, S. A.,et al.[4] determined relationships between the discharge coefficient and the ratio of head on and radius of morning-glory spillway. AKhanmeneh, H. Z., et al.[5] studied the effect of semi-circular side weirs on hydraulic properties and discharge coefficient. They proved that the semi-circular weir give high coefficient value when comparison with rectangular weir. Kumar, S., et al.[6] studied experimentally the discharge coefficient of sharp crested weir of curved plan-form. They found that gain about 40% in discharge over curved weir with vortex angle 90^{o}. Severi, A., et al.[7] investigated experimentally discharge coefficient changes of moveable cylindrical weir gate. Bagheri, S.and Heidarpour, M.[8] studied overflow characteristics of circular crested weir to determined discharge coefficient and velocity values over crest with an rotational vortex. Vatankhah, A. R.[9] presented equation to flow over circular crested sharp weir to measure flow discharge. Gonzalez, J. A.[10], Damisse, E., et al.[11] discuss the rating for pressure flow in gated culverts with weir-box inlet. Castro-Orgaz, O., et al.[12] examined experimentally simplified models incorporating streamline curvature effects for critical flow over circular crested weirs.

EXPERIMENTAL SETUP

The flume used in this work is of relatively large-scale size (30 cm bed width). It is rectangular and re-circulating flume with a closed operating system. The overall length of the flume is 15.6 m, which is divided into three parts which are the inlet, the outlet and the working section of the flume. The length of the inlet, working section and the outlet parts is 1.7, 12.5 and 1.4 m respectively. A general view of the flume is shown in figure 1.

The sump tanks consist of five tanks, which are made from fiberglass. The first tank has dimensions of 1.15 m long, 0.6 m wide and 0.85 m deep. The remaining tanks have the same dimensions (2.25, 0.7, and 0.6) m. The first tank is connected to the second tank by flexible rubber connection. The other tanks are connected to each other by P.V.C pipes with diameter of 0.25 m through rubber connections. The first and last tanks have a drain valve at the lower part of their sides to drain the water of the sump tanks when required. The flow system is a closed circuit as the flume is re-circulating type. The flume takes its water from the sump tanks through a 0.1m-feeding pipe, which is connected to the pump. The tail gate consists of an aluminum plate, which is fixed at the end of the working section. Rubber sheets at its both sides are provided to prevent leakage at its connections with the outlet sides. The gate is pivoted at the bottom of the outlet tank through a rubber connection to be watertight. The submerged flow can be obtained by controlling the tail gate opening. The tail gate can be moved by using a hand driven gear system fixed at the top of the outlet tank.

The tested models are vertical circular weirs have diameter 10.16cm, 12.70cm and 15.24cm with different heights 29cm, 31.5cm and 34cm. The pipe culvert fixed at depth 10cm from channel bottom and its diameter 7.6cm. Pipe hole with diameter 2.6cm fixed at 6cm from weir bottom level in upstream. The channel flow discharges measured ranged between 0.5l.t/sec to 6.5lit/sec . The number of experimental run for the research is 300 runs. Each run measure upstream and downstream water depth, water depth above weir, water depth inside weir, and flow discharges at different weir heights and diameters. Also all data collected at weir only and at combined weir with upstream pipe hole.

Figure 1 Experimental work arrangement

MATHEMATICAL APPROACH

Different parameters as shown in fig.2 are control for flow criteria and characteristics. Its parameters are channel bed width b (L), circular weir diameter D_w (L), circular weir heights P (L), head above weir H_w (L), Pipe culvert diameter d_c (L), head above culvert h_{iw} (L), diameter of pipe hole at upstream weir d_o (L), head above pipe hole in upstream weir h_o (L), upstream water depth $h_{u.s}$ (L) , downstream water depth $h_{d.s.}$ (L), flow velocity v($L.T^{-1}$), and flow discharge Q ($L^3.T^{-1}$). The fluid characteristics are fluid density ρ (M/L^3), dynamic viscosity μ ($F.L^{-2}.T$). Apply continuity equation, energy equation, and momentum equations [16,17] to find the discharge coefficient equations from measurements collected data..

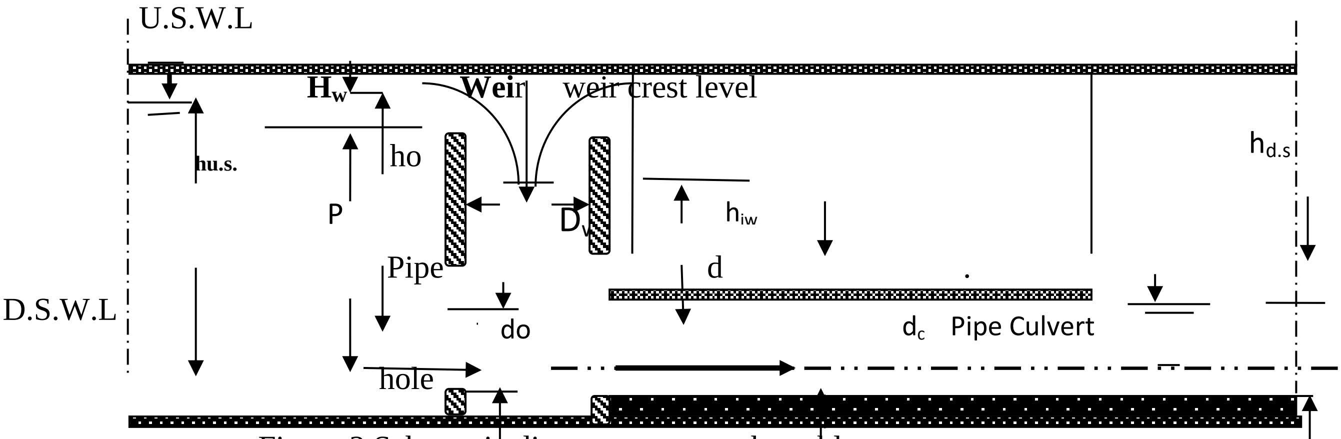

Figure 2 Schematic diagram to research problem

This structures known by tail escapes, its combined between circular weir, pipe hole in upstream and pipe culvert in downstream. By applying first principle of fluid mechanics and open channel theories, we deduce the the discharge equations for each structures and at combined.

Discharge coefficient of circular weir

For circular weir[8],

$$Q_w = \frac{2}{3}.c_{dw}.\pi.D_w.\sqrt{2.g}.H_w^{1.5} \quad (1)$$

$$C_{dw} = \frac{3.Q_w}{2.\pi.D_w..\sqrt{2.g}.H^{\frac{3}{2}}} = 0.0344 * \frac{Q_w}{D_w*.H^{1.5}} \quad (2)$$

Discharge Coefficient of downstream pipe culvert

French, and Chow[13& 14] discuss the different pipe culvert equations at different flow condition and find that:

$$Q_c = C_{dc} * a_c * \sqrt{2g * (h_{iw} - h_{d.s})} \quad (3)$$

$$C_{dc} = \frac{0.0918*Q_c}{d_c^2*\sqrt{h_{iw}-h_{d.s}}} = \frac{0.0918*Q_c}{d_c^2*h_c^{0.5}} \quad (4)$$

In which= h_{iw}-$h_{d.s}$=h_c

Discharge Coefficient of Upstream Pipe Hole

Pipe hole is known by orifice meter, the discharge equation can be written as follow,

$$Q_o = C_{do} * a_o * \sqrt{2g * h_o} \quad (5)$$

$$c_{do} = \frac{0.0918*Q_o}{d_o^2*h_o^{0.50}} \quad (6)$$

For Case of Weir and culvert without pipe hole

From continuity equation, Q_w=Qc, then

$$\frac{2}{3}.C_{dw}.\pi.D_w.\sqrt{2.g}.H_w^{1.5} = C_{dc} * a_c * \sqrt{2g * h_c} \quad (7)$$

$$C_{dw}=0.375 * \mathrm{C_{dc}} * \left(\frac{\mathrm{d_c^2}}{\mathrm{D_w}}\right) * \sqrt{\frac{h_c}{H_w^3}} \quad (8)$$

Combined between circular weir, upstream pipe hole and downstream pipe culvert

For continuity equation the total discharge equal the summation of upstream pipe hole and circular weir and equal the discharge of downstream pipe culvert.

$$Q_{meas} = Q_o + Q_w \quad (9)$$

In which Q_{meas} is the measured discharge, then

$Q_{meas} = Q_c = C_{dav} * (Q_o th + Q_w th)$, in which Q_oth, and Q_wth are theoretical discharge of weir and orifice respectively.

$$C_{dav} = \frac{Q_{meas}}{(a_o*\sqrt{2g.h_o}+a_c.\sqrt{2g.h_c})} \quad (10)$$

In which C_{dav} is the discharge coefficient in combined flow cases.

DIMENSIONAL ANALSIS

The relationship between the flow discharge over circular weir, upstream pipe hole and downstream pipe culvert and the other dependents parameters can be deduced by dimensional analysis. The dimensional analysis relationship used to check the sensitivity of the different parameters which affect on the above equations. The general function can be written as follows;

$$f(Q, \rho, g, \mu, v_o, P, D_w, H_w, b, h_{d.s}, h_{u.s,}, d_c, d_o, h_o, h_c) \quad (11)$$

Using the dimensional analysis Buckingham's π theory in which ρ, H_w, and v are selected as repeated variables representing fluid properties, pipe geometry characteristics and the flow characteristics respectively, then:
The discharge coefficients can be written as follows;

$$C_{dw} = f\left(\frac{b}{D_w}, \frac{H_w}{P}, \frac{H_w}{D_w}, \frac{h_{u.s.}}{P}\right) \quad (12)$$

$$C_{do} = f\left(\frac{h_o}{d_o}, \frac{H_w}{P}, \frac{h_{u.s.}}{d_o}\right) \quad (13)$$

$$C_{dc} = f\left[\frac{h_{u.s.}-h_{d.s.}}{d_c}, \frac{h_c}{d_c}, \frac{H_w}{d_c}, \frac{D_w}{d_c}, \frac{h_o}{d_c}\right] \quad (14) \text{ and}$$

$$C_{dav} = f\left[\frac{H_w}{P}, \frac{h_c}{d_c}, \frac{h_o}{d_o}\right] \quad (15)$$

RESULTS ANALYSIS AND DISCUSSION

Relationship between C_{dw}, and (H_w/D_w)

Figure 4 shows the effect of the circular weir heights on the weir discharge coefficient. The relationship between C_{dw} and (H_w/D_w) indicated that the weir discharge coefficient decreases as the ratio (H_w/D_w) increases and increases as the weir heights increases. We noticed that the average increases in weir discharge coefficient is 67% as the percentage in weir heights increases by 14.7%.

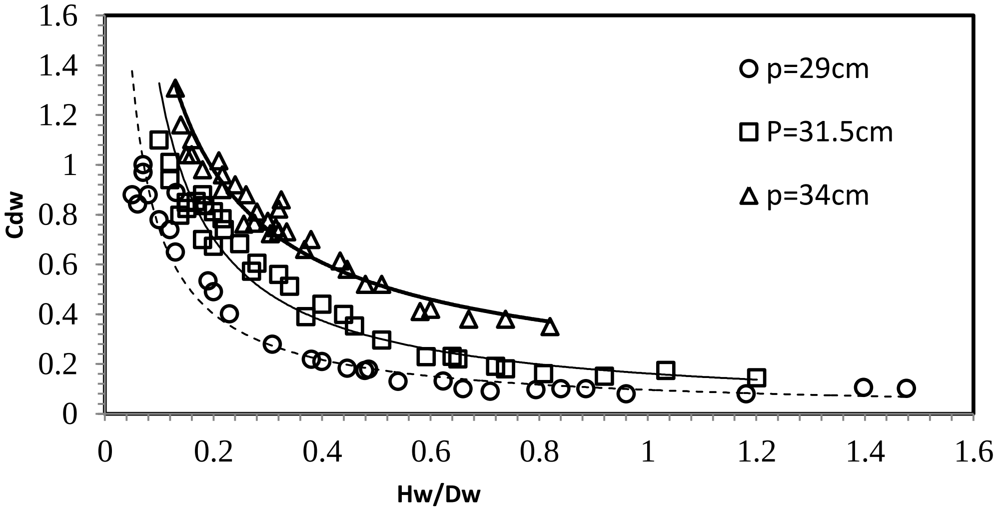

Fig.4 Relationships between (Hw/Dw)and Cdw at different weir heights.

Relationship between C_{dw}, and (H_w/P)

Effect of weir diameters on the circular weir discharge coefficient at different ratio of (Hw/p) shown in figure 5. It indicated that the circular weir discharge coefficient decreases as the ratio of (Hw/p) and weir diameters D_w increases . The average decreases in C_{dw} is 42% as the increases in weir diameter by 32%. This means that the increases in weir diameters causes decreases efficiency of weir and also the increases in H_w/p above 0.1 causes decreases in weir efficiency.

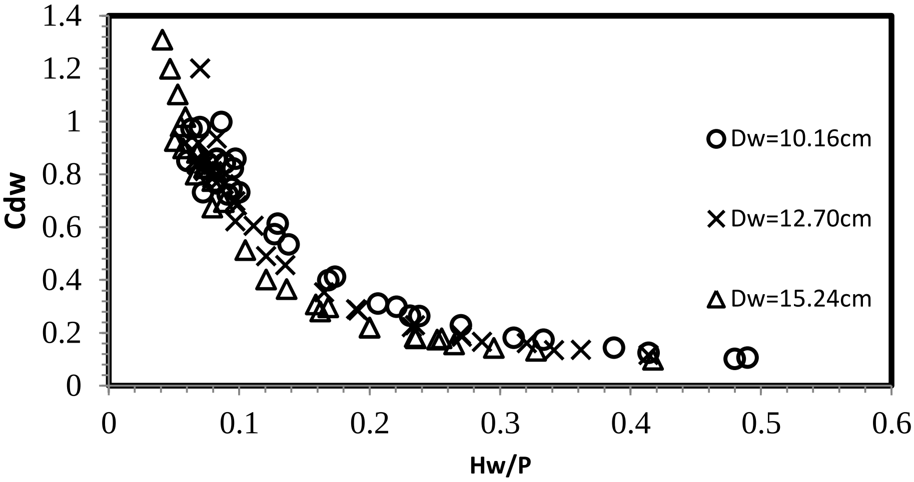

Fig. 5 Relationship between Cdw and Hw/P at differents circular weir diameter

Relationship between head - discharge for circular weir

Figure 6 shows the relationship between weir discharge (Lit/sec) and head above weir H_w(cm) to trace effect of weir heights and diameters on the weir discharges. The figure indicated that the increases in weir heights have big effect when compared with increases in weir diameters. The minimum flow discharges occurred at weir diameter =10.16cm, and weir height=29cm. The co-relation between head and discharge can be written as follows,

$$Q_w = 1.037 * (H_w)^{1.377} \quad , R^2=98.4\% \qquad (16)$$

Also the maximum circular weir flow discharge occurred at weir diameter =15.24cm, and weir heights=34cm. The regression analysis to co-relate between H_w and Q_w can be written as follows;

$$Q_w = 2.074 * (H_w)^{1.42} \quad , R^2=98.4\% \qquad (17)$$

All measured data closed between the two above equations.

The variation in the weir flow discharge depend on weir diameter and height

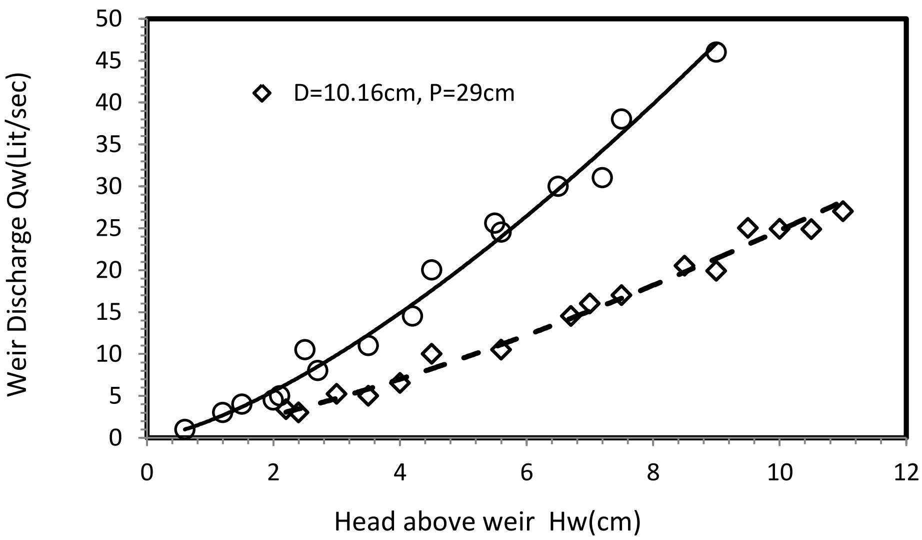

Fig. 6 Discharge head relationship for ciircular weir at different heights and weir diameters

Relationship between C_{dc}, and (h_c/d_c) for downstream pipe culvert

Tail escapes have downstream culvert fixed in circular weir to remove the weir discharge to downstream water ways. Figure 7 shows the relationship between culvert discharge coefficient C_{dc} and the ratio of head above culvert to culvert diameter (h_c/d_c) for free flow. The figure indicated that the discharge coefficient decreases as the ratio of (h_c/d_c) increases. The regression analysis of experimental data at all flow cases can be written as follows,

$$C_{dc} = 0.784 - 0.069 * (h_c/d_c) - 0.009(h_c/d_c)^2 \quad , R^2=94.7\% \qquad (18)$$

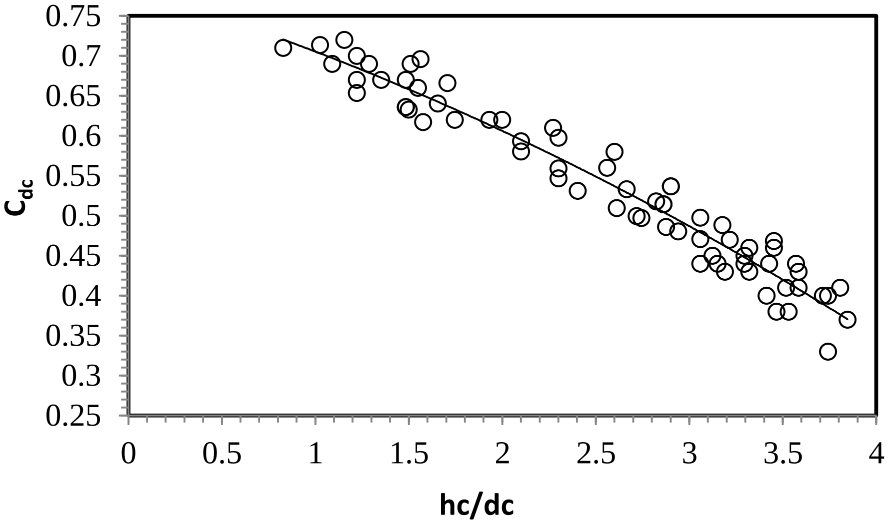

Fig. 7 Relationship between C_{dc} and h_c/d_c

Relationship between head- discharge for downstream pipe culvert

Figure 8 shows the relationship between h_c and Q_c for downstream pipe culvert. The pipe culvert flow discharge increases as the head increases. The regression analysis between Q_c and h_c can be written as follows,

$$Q_c = 1.204 * (h_c)^{0.47} \quad ,R^2=90.6\% \qquad (19)$$

For maximum flow of weir at H_w=12cm, then Qw=6.39lit/sec, for continuity apply in equation 18 the pipe culvert flow discharge =6.00lit/sec, then the maximum difference error is 6.1%. For minimum flow Qw=2.12Lit/sec, and Qc=2.01lit/sec , then the minimum error is 5.2%. Then the error in measured range between 5.2% and 6.1%.

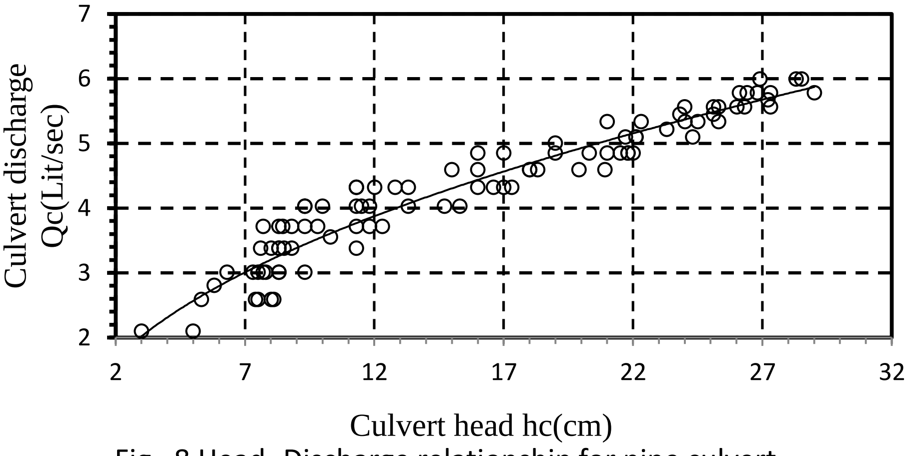

Fig. 8 Head- Discharge relationship for pipe culvert

Relationship between C_{do} and (h_o/d_o) for upstream pipe hole

The relationship between discharge coefficient of upstream bottom pipe hole in upstream circular weir and ratio of (h_o/d_o) shows in figure 9. The equation of computed C_{do} can be written as follows,

$$C_{do} = 0.098 * (h_o/d_o) - 0.582 \quad , \quad R^2=99.8\% \qquad (20)$$

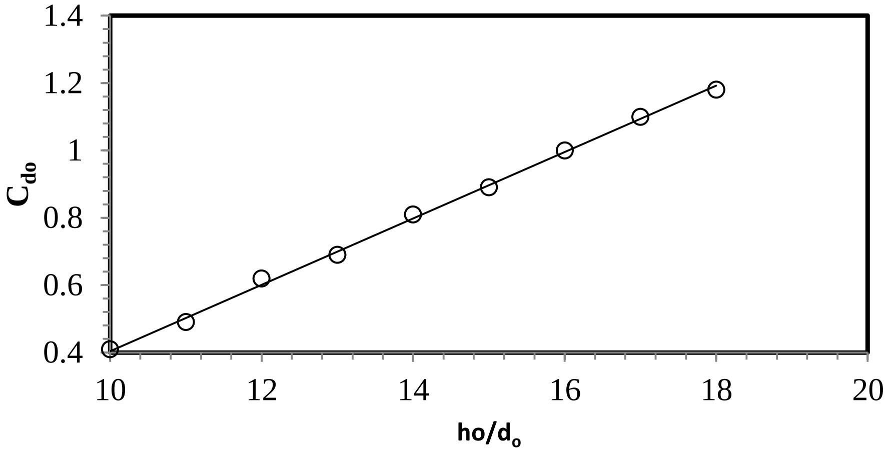

Fig. 9 Relationship between C_{do} and(h_o/d_o)

Relationship between Head- Discharge for upstream pipe hole.

Figure 10 shows the head discharge relationship for pipe hole at bottom level of circular weir. The discharge increases as the head increases and correlation regression analysis can be written as follows;

$$Q_o = 0.207 * (h_o)^{0.473} \qquad R^2=98\% \qquad (21)$$

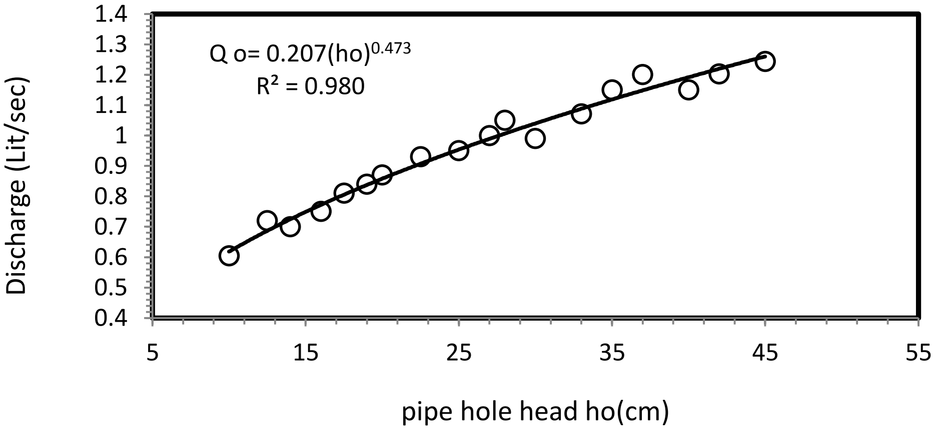

Fig. 10 Relationship between pipe hole discharge and head

Combined between upstream pipe hole, and Circular weir

Relationship between C_{dav}, C_{dw}, , C_{dc}, C_{do} and H_w/p, h_c/d_c, h_o/d_o at different weir diameters

Figures 11, 12 &13 show the relationship between discharge coefficients for combined circular weir, pipe hole, pipe culvert and average combined versus ratio (H_w/P), $(h_c/d_c)$and (h_o/d_o). It indicated that the circular weir have big effect on the average discharge coefficient. The ratio of $(H_w/p=0.1)$, $(h_c/d_c=1.5)$ and $(h_o/d_o=10)$, is the optimum value give equal value of discharge coefficient =0.60. For increases in (H_w/P, h_c/d_c and h_o/d_o) cause decreases from weir discharge coefficient, average discharge coefficient, little deacreases culvert discharge coefficient and increases discharge coefficient of upstream pipe hole in upstream. Table 1 show the correlation of regression analysis of the experimental data for different regression equations:

For upstream pipe hole,

$$C_{do} = a_1 * (F) + b_1 \quad (22)$$

For circular Weir, downstream pipe culvert, and combined case can be written in the form,

$$C_{dw}, C_{dc}, C_{dav} = a_1 * (F)^{b_1} \quad (23)$$

The coefficient a_1, b_1, b_2 and F are tabulated in table 1. F=(Hw/p), (hc/dc) or (ho/do)

Table 1 Coefficient of regression analysis of co-relation equations

Model Type	a_1			b_1			R^2 %		
	(Hw/P)	(hc/dc)	(ho/do)	(Hw/P)	(hc/dc)	(ho/do)	(Hw/P)	(hc/dc)	(ho/do)
circular pipe hole	1.361	0.223	0.012	0.446	0.187	1.687	98.6	93.1	92.5
Circular weir	0.025	1.655	$3*10^6$	-1.27	-2.34	-6.75	93.6	98.1	90.0
pipe culvert	0.492	0.71	3.906		-0.29	-0.79	86.0	89.1	90.1
Combined model	0.061	1.375	68498	-0.96	-1.71	-5.06	95.4	97.5	86.7

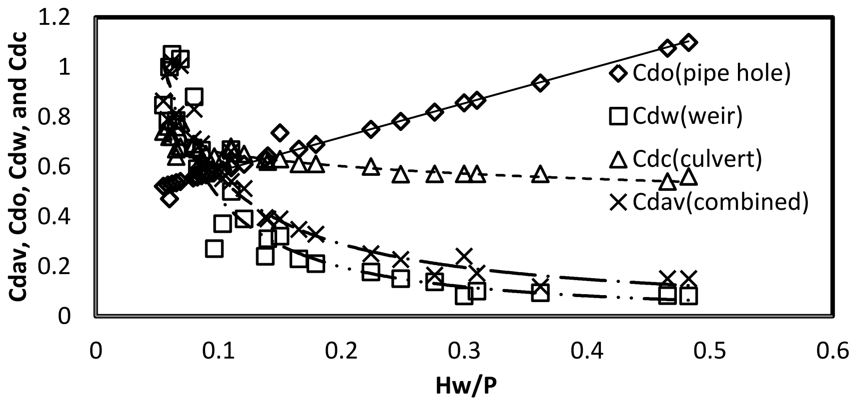

Fig. 11 Relationship between Cdav, Cdo, Cdw, and Cdc and Hw/P at different weir diameters and p=29cm

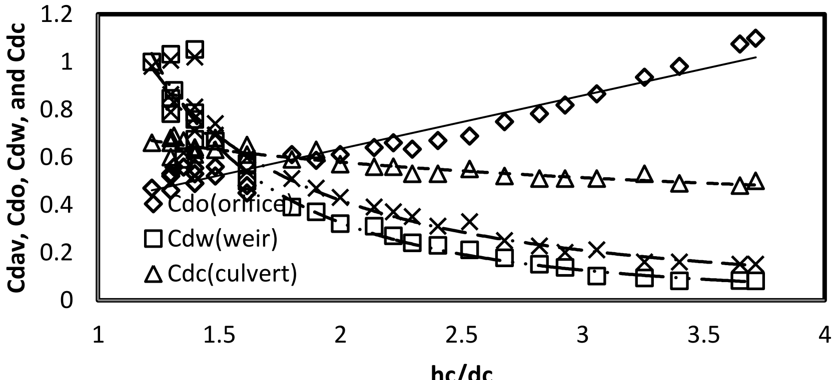

Fig.12 Relationship between Cdav, Cdo, Cdw, and Cdc and hc/dc at different weir diameter and p=29cm

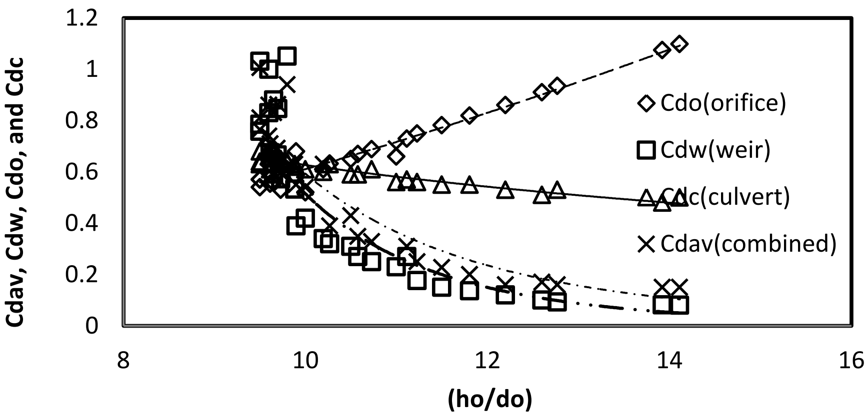

Fig. 13 Relationship between Cdav, Cdw, Cdo, Cdc and (ho/do) at different diameters and p=29cm

Ratio of weir discharge Q_w, upstream pipe hole discharge Q_o to total channel discharge Q_T

Figure 14 show the relationship between Qo, Qw, QT and upstream water depth (hu.s.) in case of combined structures. It indicated that the weir have big effect on total discharge. The total discharge curve take parrale shape to weir discharge curve. Figure 15 show the percentage of Q_o, Q_w to Q_T as a percentage. The co-relation between Qo/Q_T and Q_w/Q_T as shown in this figure and can be written as follow,

$$\left(\frac{Q_w}{Q_T}\right) = 100 - \left(\frac{Q_o}{Q_T}\right) \tag{24}$$

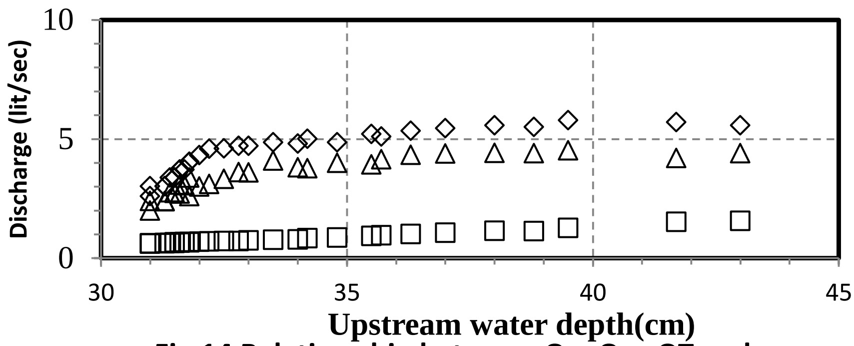

Fig.14 Relation ship between Qo, Qw, QT and Upstream water depth (hu.s.)

◇ Total Discharge QT □ Upstream pipe hole discharge Qo
△ Circular weir discharge Qw

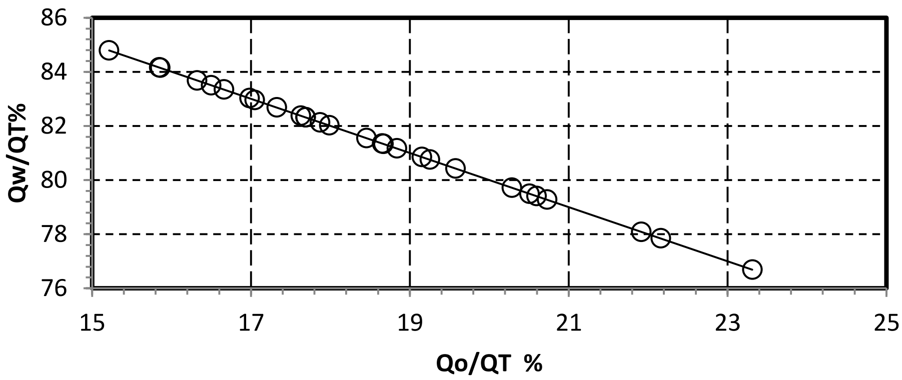

Fig. 15 Relationship between Qo/QT and Qw/QT

Developed Empirical Equations to compute Discharge Coefficients

From figure 3, 4 and dimensionless equation 14 we deduced the developed equation and written as follows:

$$C_{dw} = 0.0805 * (H_w/D_w)^{-0.91} + 0.0205 * (H_w/P)^{-1.17} \quad (26)$$

The equation is valid $0.1<H_w/D_w)<1.6$, and $0.1<H_w/P<0.6$

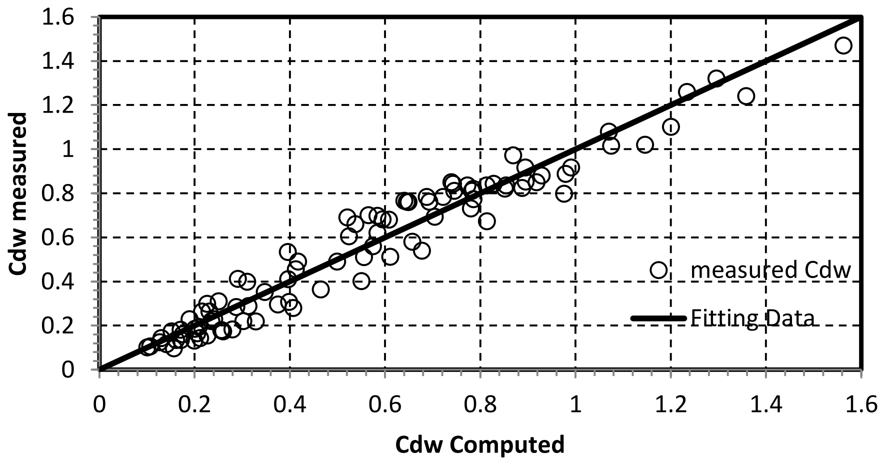

Fig. 16 Relationship between computed and measured Circular weir discharge coefficient

From equation 17, figures 11 and 12 the developed equation to computed the combined discharge coefficient in function of Hw/p and ho/do.

$$C_{dav} = 0.0305 * (Hw/p)^{-0.97} + 34249 * (h_o/d_o)^{-5.06} \quad (27)$$

For 10<ho/do<15, 0.1<Hw/P<0.5

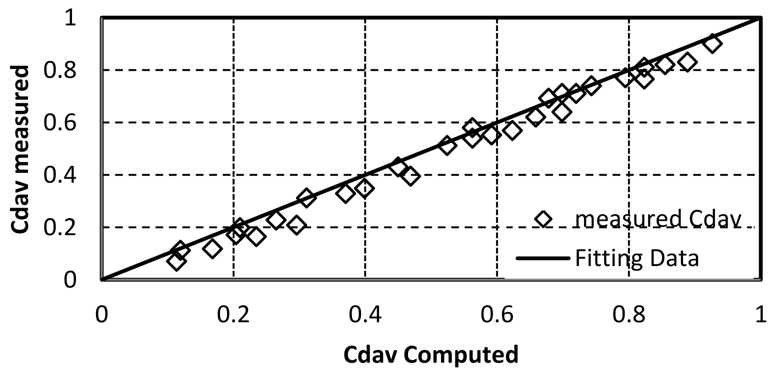

Fig. 16 Relationship between Cdav computed and measured for combined weir and hole pipe

CONCLUSIONS

The effect of vertical circular weir heights and diameters on the discharge efficient of weir, upstream pipe hole and downstream pipe culvert are studied. The main conclusions can be summarized as follow.

1- The vertical circular weir discharge coefficient increases as the weir height increases and decreases as the increases in weir diameters.
2- Doveloped head – discharge equations to compute circular weir flow discharge, culvert discharge and pipe hole discharge.

3- For combined vertical circular weir and upstream pipe hole the optimum average discharge coefficient is 0.6 at Hw/p = 0.1 and ho/do = 10 .
4- For combined structure the weir have big effect due to the weir discharge not less than 72% from total discharge.
5- New statistical Analysis equations developed to compute vertical circular weir discharge and average combined discharge coefficient and error not more than 6% .

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