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USING NONLINEAR FINITE ELEMENT AND YIELD LINE THEORY IN SOLVING TWO WAY SOLID SLABS UNDER CONCENTRATED LOADS

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ملخص البحث

تعتبر البلاطات المصمتة واحدة من اكثر الانظمة الانشائية شيوعا فى الاستخدام. البلاطات المصمتة هى بلاطات خرسانية ذات سمك ثابت تنقل الأحمال الى الكمرات المحيطة بها.
المشكلة التى دائما ماتوجهنا هى انه قد نحتاج فى كثير من الاحيان الى اضافة فتحات فى هذه البلاطات و ذلك لتركيب العديد من الاشياء مثل السلالم الداخلية او المصاعد او السلالم الكهربائية و فتحات للتوصيلات الكهربائية او ا لمواسير او للتكييف. فى هذه الحالة و خصوصا اذا كان التركيب يتم فى البلاطات امصبوبة مسبقا لا يمكننا التأكد من قدرة البلاطات على تحمل الأحمال مع وجود الفتحات الجديدة وكذلك لا يمكننا التأكد من مقوماتها للترخيم. من المعروف انه توجد طريقتان فقط لزيادة قدرة البلاطات الخرسانية على تحمل الأحمال الخارجية (زيادة سمك البلاطة -اضافة حديد تسليح اضافى حول الفتحة) فى هذا البحث نستعرض الاختبارات العملية لثلاث بلاطات ؛ تمت مقارنة النتائج المعملية مع النتائج النظرية و ذلك باستخدام نظرية خطوط الكسر و التحليل الخطى و كذلك باستخدام برنامج ANSYS.

ABSTRACT:

Slabs are one of the most important components in any building. They are structural elements which are designed intrinsically by techniques based upon the elastic theory. A method for slab analysis is presented as a practical substitute to full, non-linear, finite element methods that require expert knowledge and long running times. The method provides a general, safe and efficient way to analyze reinforced concrete slabs up to failure. Yield line analysis is an equivalent for two dimensional flexural members (plate or slab) of limit analysis of a one dimensional member (continuous beam). In this research, yield line analysis was used to calculate the maximum loads for two way solid slabs with openings. Also, linear finite element program (SAP2000) and nonlinear finite element program (ANSYS) were used to analyze tested specimens. The effect of the area of steel on the maximum load was analyzed using ANSYS.

In the experimental Program, two way solid slabs with dimensions (1650mm×1650mm×80mm) were surrounded by beams of dimensions (100mm×250mm). Central openings of dimensions (300mm×300mm) were contained and introduced to different specimens. The slabs were loaded using a four point load system. Results showed that the concentrated load caused variation in crack patterns. The yield line theory gave maximum loads compared to the experimental. For ANSYS, minimum steel reinforcement proved to be essential for the ductility of the reinforced concrete slab.

Keywords: yield line analysis, non linear finite element analysis, two way solid slabs, openings.

1. INTRODUCTION

Introducing openings to slabs before or after casting affects slabs' load carrying capacity and their ability to resist deflection and service loads; so designers should have a solution to manage such losses and to make up for them and take precautions during the design stage. Concerning the last matter yield line theory was used suggesting different crack patterns to calculate the limit load for slabs with openings.

In the design of concrete structures handbook (chapter 23) [1] , it is stated that yield line analysis is founded upon the principle of conservation of energy: the work performed by an external force moving through a distance is equal to the internal work performed by rotations about plastic hinges that resist the external force. The yield line analysis method provides an upper limit estimate of the maximum ultimate resistance of a slab for an assumed mode of failure.

The term 'yield-line' was coined by Ingerslev [2]. Gvozdev [3] determined the value of the collapse load for statically indeterminate systems undergoing plastic deformation. Johansen [4] first proposed yield line, he solved many problems pertaining to the ultimate strength of reinforced concrete slabs subjected to uniformly distributed or point loads. Throughout the work of Nielsen [5] and Jones and Wood [6] yield line analysis of reinforced concrete slabs was introduced into the wider structural engineering community. As the last two outlined the necessity of the membrane analysis and the serviceability criterion for designing slabs.

Prager [7] explained the general concepts of plasticity, which comprise the general multi-axial stress-strain relations, normality and convexity, maximization of plastic energy dissipation, limit state theorems, shakedown, optimum design, plastic hinges, yield line theory of plates and slip line theory. Nielsen, M.P. [8] illustrated concrete plasticity of slabs, yield condition orthotropic slabs, bi-conical yield surface and arbitrary reinforcement.

Coulomb, C.A. [9] clarified his own failure criterion. Braestrup M. U. et al. [10] gave an exact plastic Solution for beams without shear reinforcement by assuming that the materials were perfectly plastic.

Mansur & Tan [11] proposed analysis and design procedure for beams with circular and rectangular openings. The analytical model proposed is able to deal with combined bending, shear and torsion in beams with openings, and the reinforcements required for this combined action. The proposed analysis and design procedure are not applicable to reinforced concrete slabs. Park & Gamble [12] conducted a review on analysis of reinforced concrete slabs with openings and reported that an opening in a simply-supported square slab with dimension of 0.2 to 0.3 times of the slab dimension could cause a reduction of 11% in the ultimate load per unit area. Larger opening with dimension of 0.5 or more times the slab dimension would not result in reduction of ultimate load per unit area.

El-Salakawy et al. [13] tested six full-scale reinforced concrete slabs, of which five were slabs with various arrangements of openings in the vicinity of the column. The openings were square with sides parallel to the sides of the column; one opening had the same size as the column and the other is 60% of the column size. Both openings led to reduction in ultimate strengths of the slab by 30% and 12% respectively. Teng et al. [14] experimented full scale slabs with openings. It was reported that openings reduced the punching shear strength of slabs considerably, and the recommended locations for openings in slabs are along the longer side of a column.

According to different Codes of practice, the minimum reinforcement ratios for mild steel ranges from 0.0015 bt_s to 0.0025 bt_s for Indian standard (IS 456: 2000) [15] and Egyptian code of practice ECP (203–2007) [16] respectively. According to ACI Code7.12.2.1.[17], and UBC Code [18], reinforcement ratio was said to be not less than 0.14% for shrinkage and temperature stresses normal to flexural reinforcement shall provide in structural slabs for flexural ratio not less than 0.2%.

Gawas S. and Itti Dr. S.V. [19] used ANSYS to model two way reinforced concrete slabs with and without openings to understand the behavior of slab with different boundary conditions. The study showed that the displacement is highest in slab having simple support on all sides and stresses were least in same slab along the edges. Also slab with fixed support on all sides shows least displacement and highest stresses along the edges of the slab.

Mohan R. T. and K. M. S. [20] investigated the structural behavior of two way reinforced concrete slab with and without openings for different slab length ratios and different opening ratios. Those different models of slab with and without opening were modeled in finite element software ANSYS. It was concluded that in all cases the least value of reduction in strength is for opening with length to breadth ratio 2 and highest value is for square slabs, which is when length to breadth ratio is one. In case of slab with all four edges continuous, the reduction in ultimate strength is about 0 to 6% for opening area 20 to 30% of slab area.

2. SIGNIFICANT RESEARCH

Life cycle is accelerated and changes are taking place in order to keep pace with those variations. Steps to execute different procedures are getting much easier so people need to switch many things roles including facilities, machines and even buildings. Buildings face many tremendous changes due to sudden change in their functions. This may appear in the form of introducing openings in different places in order to situate those buildings for different uses. In this paper, the effect of the introduced openings in existing buildings under the service load was studied using yield line analysis. A comparison was held between the results from experimental work, linear finite element using SAP2000, yield line theory and nonlinear finite element analysis using ANSYS program.

3. EXPERIMENTAL DETAILS

3.1 Description of Test Specimens and Studied Parameters

The experimental program consisted of three square RC slabs with dimensions 1650mm×1650mm and 80mm thickness, they represented a group to study the effect of openings created before (B) or after (A) casting with no extra strengthening around the opening to make up for the lost strength.

Table (1): Tested specimens**.**

Slab	Opening	Strengthening Method
S1	No	No
S2	B	No
S3	A	No

All RC slabs, as shown in figure 1, are surrounded with RC beams in all directions with dimensions (100 mm x 250 mm), with two top and bottom steel bars 10mm, and 6∅8/m' stirrups.

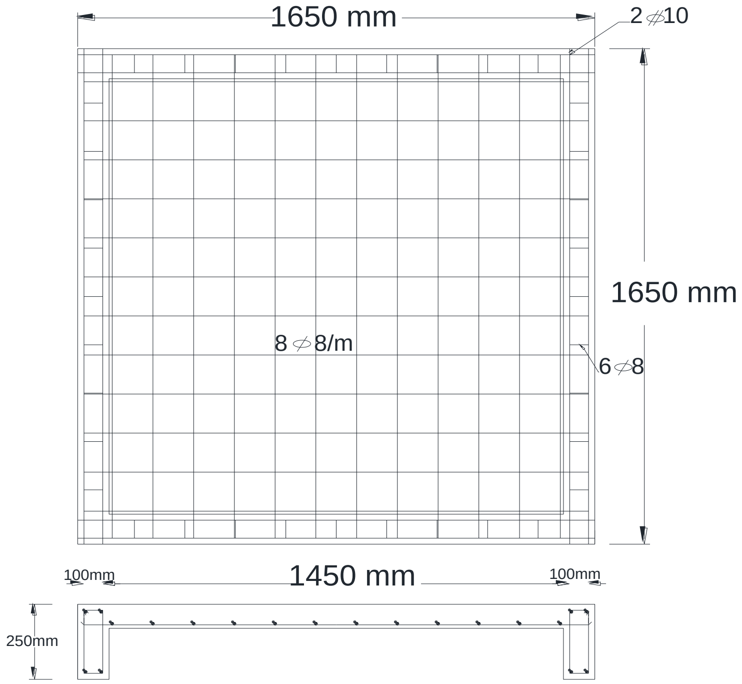


Figure (1): Typical dimensions and reinforcement of specimens without openings (S1 and S3).

3.2 Material Properties

The material properties of the concrete were determined for the slabs. The average cube concrete compressive strength after 28days (f_{cu}) was 32.6MPa, and modulus of elasticity (E_c) was 25.12GPa. The steel used for slab reinforcement and stirrups in this research was mild steel (24/35), it had 244MPa yield stress. The tension and compression reinforcement in RC beams are high tensile steel (36/52) with diameter equal to 10mm.

3.3 Test Setup, Procedure and Instrumentation

Figure 2 shows the test setup. Prior to testing all slabs were placed on the testing frame with their tension face downwards, the deflection devices (LVDTs and dial gauges) were adjusted on their upper face. The test slabs were loaded using a four point load system using a combined load and load displacement method to transfer the load to the tested slabs.

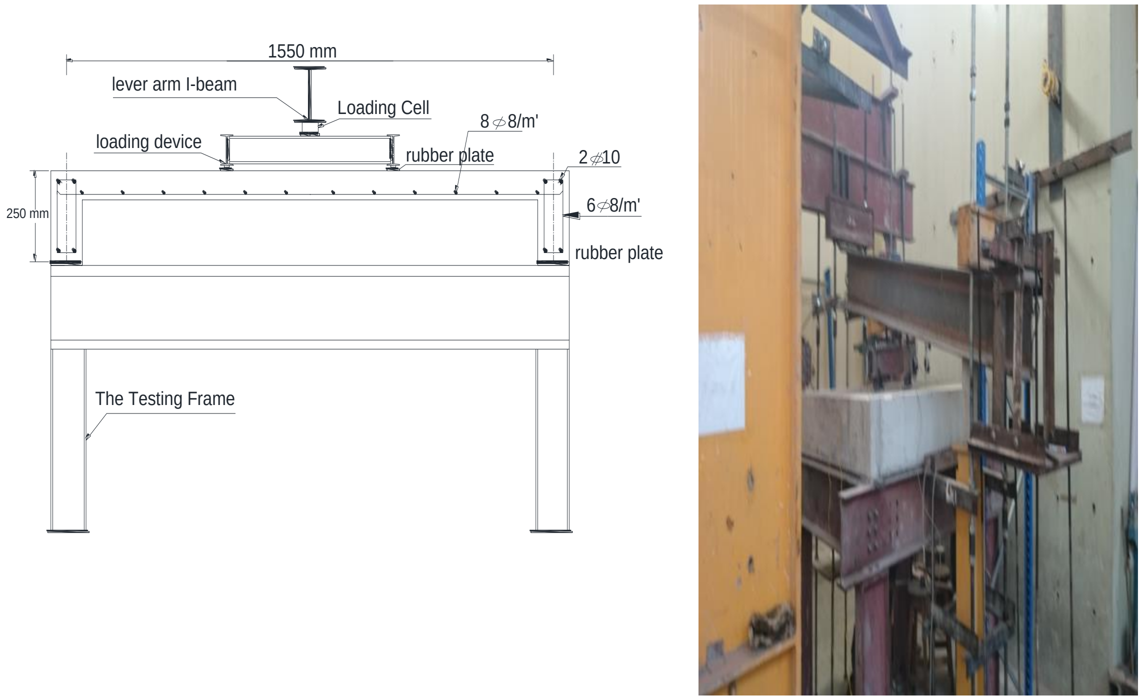


Figure (2): Test setup.

3.3. Technique of Loading and Strengthening Tested RC Slabs

For tested specimens where openings where introduced after cast certain steps were followed, as shown in figure3. 1) The slab is casted without opening. 2) The slab was loaded with one-third the load of the control slab (about 25kN). 3) The concrete cover was removed. 4) Externally steel strap of dimension 200mm×200mm was externally welded to tips of the cut steel bars around the opening (for other slabs examined the same way different strengthening was installed). 5) The steel reinforcement bars passing through the openings were cut. 6) The testing procedure continued till failure occurred.

Figure (3): Steps of introducing opening to slab (S3).

3.4 Experimental Work Results:

The load-deflection of tested specimens S1, S2, and S3 were shown in figure 5. From table (2) it is found that creating an opening before casting or after casting at service load 25kN caused a reduction in the load by 87.85% and 93.44% respectively compared to the control tested specimen. Also by comparing specimen S2 and S3, it was found that creating an opening under loading caused an increase in the deflection value.

The introduction of the opening reduced the strength of the slab. The lower value of the failure load of S2 compared to S3 indicates that the welding of the steel strips before cutting the steel bars partially maintained the pattern of moment distribution of S1 in S3 even after introducing the opening. Therefore, the behavior of the slab was midway between S1 and S2. The load carrying capacity of S3 decreased by 7% compared to the control slab and was higher than S2 by 5.6%.

Table 2: Results of the Experimental Work.

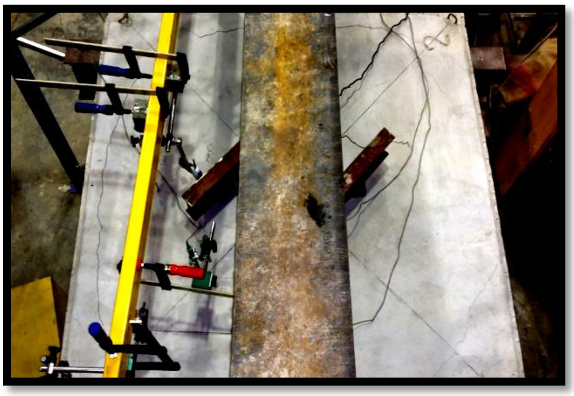
Specimens	*Opening*	P_{crak} (kN)	P_{max} (kN)	*Max. Deflection (mm)*	P_{max}/ $P_{max\ control}$	P_U/ $P_{u\ control}$ (S1)	$\delta_{specimen}$/ $\delta_{control}$
S1	No (control)	25.08	79.8	8.7	1	1	1
S2	B	25.08	70.11	13	0.878	87.85%	149.4%
S3	A	25	74.57	10.1	0.934	93.44%	116.09%

Where: P_{Cr}: the cracking load.

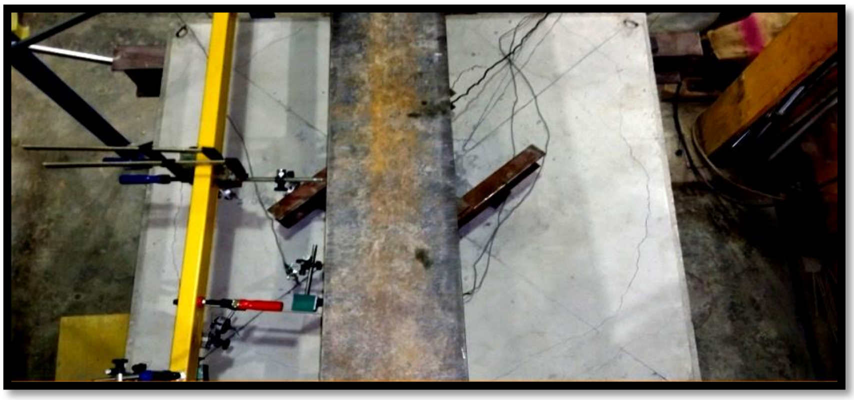
$P_{max,\ control}$: the maximum load of tested slab without opening (S1)

P_{max}: the maximum load of tested slabs with opening (S1, and S2).

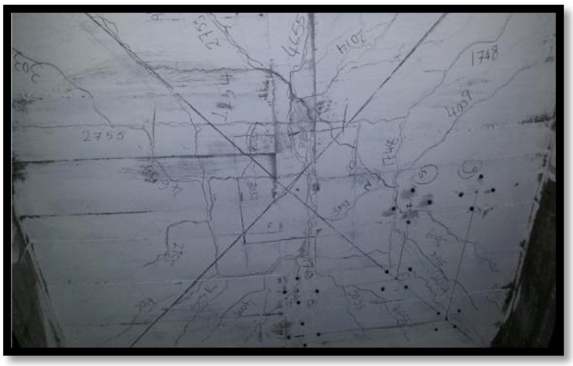
(a) Upper crack pattern of S1.



(b) Upper crack pattern of S2.



(c) Upper crack pattern of S3.



(a) Crack pattern of S1.

(b) Crack pattern of S2.

(c) Crack pattern of S3.

Figure 4: Cracks' patterns of slabs.

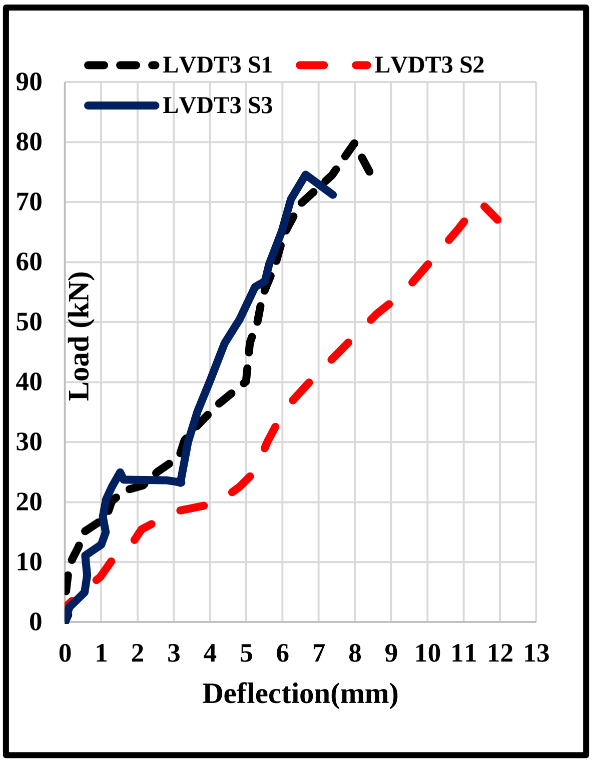


Figure 5: Load-Deflection curve of tested specimen.

4. THEORETICAL ANALYSIS

The ability of common structural analysis tools to predict the performance of the tested slabs was investigated in order to provide practicing engineers with information about their respective reliability. Analysis results of linear finite element, nonlinear finite elements and yield line theory will be discussed as follows.

4.1 Yield line analysis

Yield line theory was used to create a model to evaluate the load carrying capacity of the slabs and to investigate failure mechanisms at the ultimate limit state. The theory is based on the principle that:

Work done in yield lines rotating = Work done in loads moving.

When a slab is loaded to failure, yield lines form in the most highly stressed areas and these develop into continuous plastic hinges. These plastic hinges develop into a mechanism forming a yield line pattern. Yield lines divide the slab up into individual regions, which pivot about their axes of rotation. In order to solve the yield line analysis critical crack lines were used in the analytical model; the following steps were followed by substituting in the following equations:

$$M_{up}=M_{ux}*\cos^2\alpha+M_{uy}*\cos^2\alpha \qquad (1)$$

Where:

M_{up}: the ultimate moment of resistance along the critical crack line, M_{ux} and M_{uy}: the ultimate moment of resistance calculated in the direction of the steel reinforcement of un-strengthened concrete slabs and Angle α: the angle between the transverse axis of the slabs to a line projected from corner of the slab or from the corner of the opening if existed (α=45°).

For yield line analysis the internal energy dissipated on the yield lines during virtual rotation is equated to the external virtual work done in deflecting slab correspondingly Ue=Ui therefore

$$\Sigma P\delta=\Sigma M\theta \qquad (2)$$

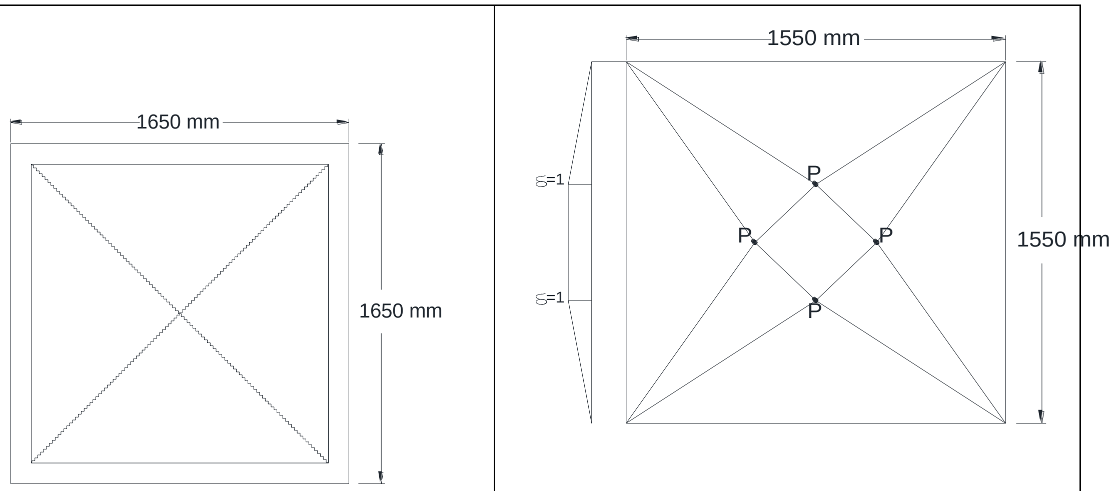
Where:

P: External Load and δ: the corresponding virtual displacement.

M is the moment defined before and θ: the angle of rotation of the slab segment.

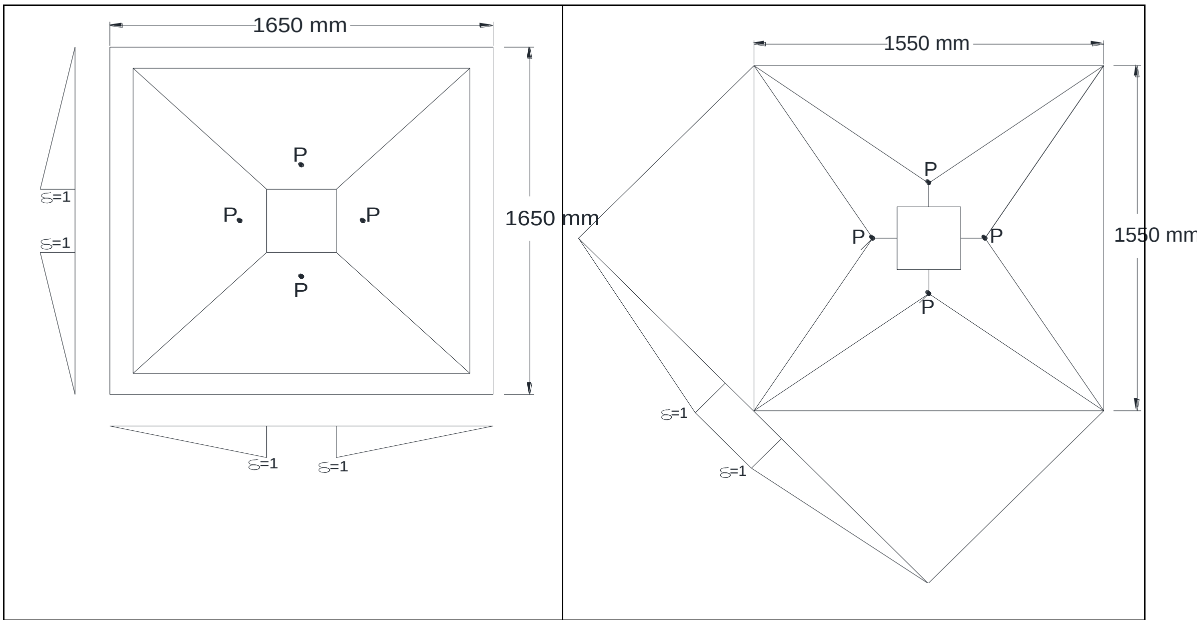
Yield lines have the following characteristics; they are straight as they represent the intersection of two planes, end at supporting edges of slabs, passes through intersection of axis of rotation of adjacent slab elements, their axis of rotation lies along line of supports and passes over columns. Yield line patterns Yield lines form under concentrated loads, radiating outward from the point of application. They also may develop from the corners of openings as they represent free edges or from the corners of slabs passing through the concentrated load.

Yield line patterns were suggested for the slab with opening introduced before or after cast taken the real cracks into consideration. The two proposed cracks patterns for two way solid slab without opening (figure 6) and two proposal crack patterns for two way solid slab without opening (figure 7) were analysis



a) Cracks' Pattern one. b) Cracks' Pattern two.

Figure 6: Proposed cracks' patterns of tested slab without opening.



a)Cracks' Pattern one | b)Cracks' Pattern two

Figure 7: Proposed cracks' patterns of tested slab with opening.

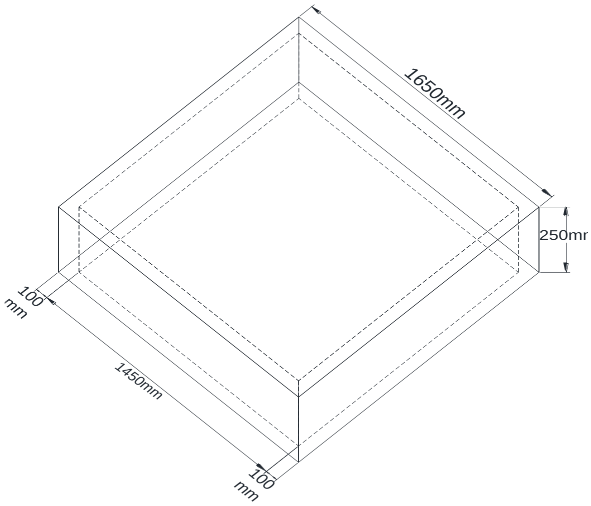
Table 3: Load carrying capacity as calculated using yield line method.

Slab	$P_{exp.(}$ kN)	Load carrying capacity of slab (kN)	
		Pattern one	Pattern two
S1	79.8	65.86	85.6
S2	70.11	52.2	80.30
S3	74.57	52.2	80.30

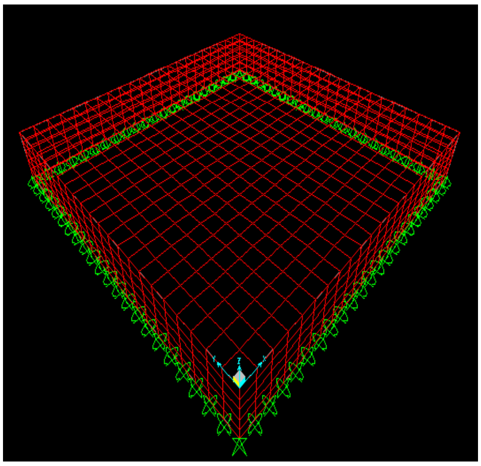
4.2 Linear Elastic Finite Element Analysis.

Linear Finite Element analysis was carried out to tested slabs by using SAP 2000. In the model, the shell elements which were used had one dimension very small compared with the other two dimensions. It carries plate bending, shear and membrane loadings. Proper boundary condition was selected as it has an important role in structural analysis. Effective modeling of support conditions at bearings and expansion joints were considered carefully as well as continuity of each translational and rotational component of displacement.

The dimensions for the slab and beams models are the dimensions of the centre planes passing at the mid thickness of the slab and the mid width of the beams as shown in figure 8.



a) Dimensions of slab and beams.



b) The mesh used of slab.

Figure 8: Dimensions of slab and beams model.

The modulus of elasticity was taken equal to $4400\sqrt{32.6}$=251122.42MPa based on the ECP203-2009 and Poisson ratio was taken equal to 0.2. The model dimensions were determined from the dimensions of the original beam-slab assembly as explained in figure 8.

Table 4: Comparison between deflection values.

Slab		Before cracking		At cracking		Maximum load	
		Practical	SAP	Practical	SAP	Practical	SAP
Slab S1	Load	12.73kN		25.08kN		79.8kN	
	Deflection	1.226	0.1724	2.415	0.3397	8.7	1.0808
Slab S2	Load	12.73kN		25.08kN		70.11kN	
	Deflection	0.7766	0.1724	1.53	0.4272	13	1.1988
Slab S3	Load	12.92kN		25kN		74.57kN	
	Deflection	1.092	0.175	2.11	0.435	10.1	1.2752

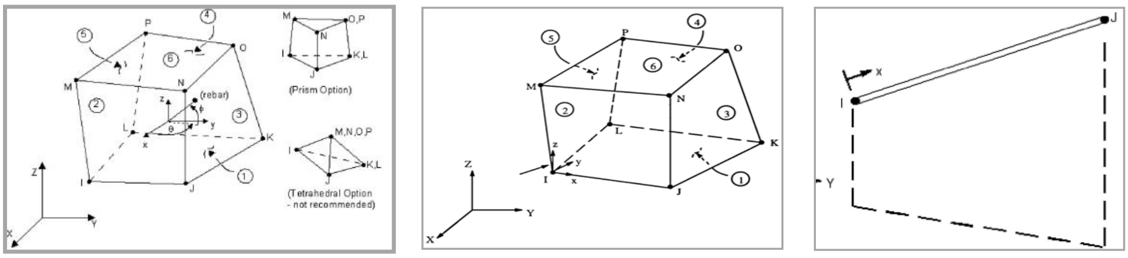
It is obvious from the previous deflection values that at lower loads they were comparable. It was found that by increasing the load the ratio between theoretical deflections to experimental decreased. By decreasing the slab thickness from 80mm to 8mm the deflections were found to be 3.74mm, 3.7811mm and 4.0026mm respectively. The deflection values did not match with practical ones due to the non-linearity resulting from cracking and yielding of steel bars or the appearance of minute cracks that were not obvious.

4.3 Theoretical Analyses Used Nonlinear Program (ANSYS)

Using nonlinear finite element program ANSYS, The reinforced concrete slabs with and without openings were analyzed to verify the theoretical models used in the experimental study. After that the effect of comparing between the slab with and without opening with different steel ratios was taken into consideration. The effect of steel ratio on the behavior of RC two way slabs was taken into consideration.

4.3.1 Modeling of steel reinforcement and concrete

A solid element, SOLID65, was used to represent the concrete element in ANSYS program. The geometry and the nodes of this element are shown in figure 9-a. the steel plates at the supports for the slabs were modeled using Solid185 elements. The geometry and the nodes of this element are shown in figure 9-b. 3D spar Link180 element is used for steel bars. The nodes and geometry of this element are shown in figure 9-c.



a) Solid 65 Element. b)Solid 185 Element. c) Link180 Element.

Figure 9: Modeling of the different elements by ANSYS program.

Equation (3), was suggested to represent the uniaxial compressive stress-strain relationship for concrete.

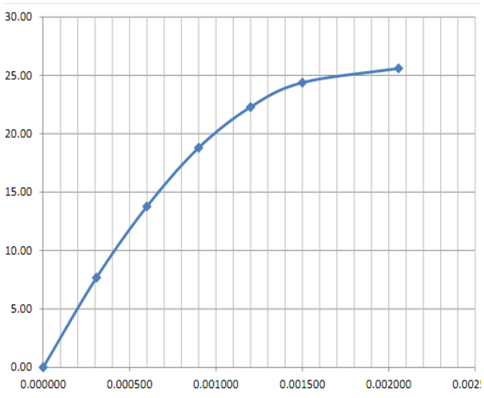
$$f = \frac{E_c.\varepsilon}{1+\left(\dfrac{\varepsilon}{\varepsilon_o}\right)^2} \qquad (3)$$

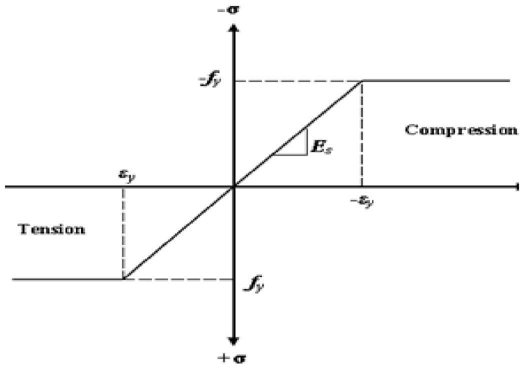
Where; f = stress at any strain ε.

ε_o = strain at the ultimate compressive strength, fc' ($\varepsilon_o = 2 f_c'/E_c$).

$f_{c'}$ = ultimate compressive strength for concrete and according to the ECP203, it can be taken equal to 0.8 $f_{cu.}$

This equation was used to plot the multi-linear isotropic stress-strain curve for concrete from 0.3fc' till ultimate compressive strength, fc', (figure 10-a). The stresses strain curves of steel bars were presented in figure 10-b. The model of the steel bars and concrete used in ANSYS program were shown in Figure 11-a, and figure 11-b respectively.

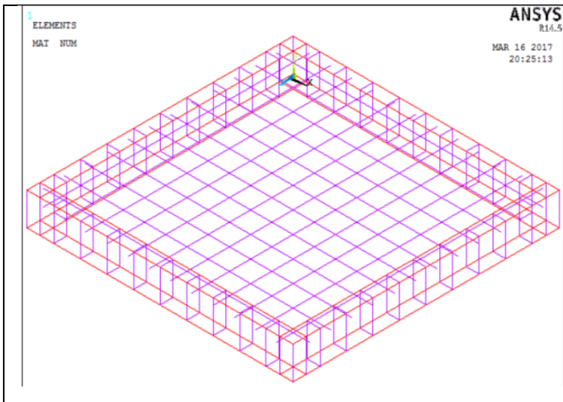




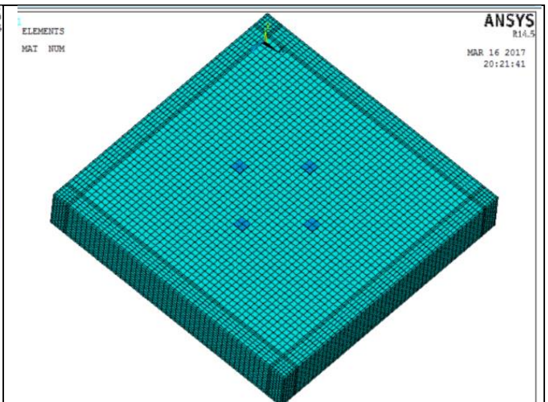
a) Uniaxial stress-strain curve for concrete.

b) Stress-strain curve for steel reinforcement.

Figure 10: the stress strain curves used in ANSYS program.



a) Steel bar model

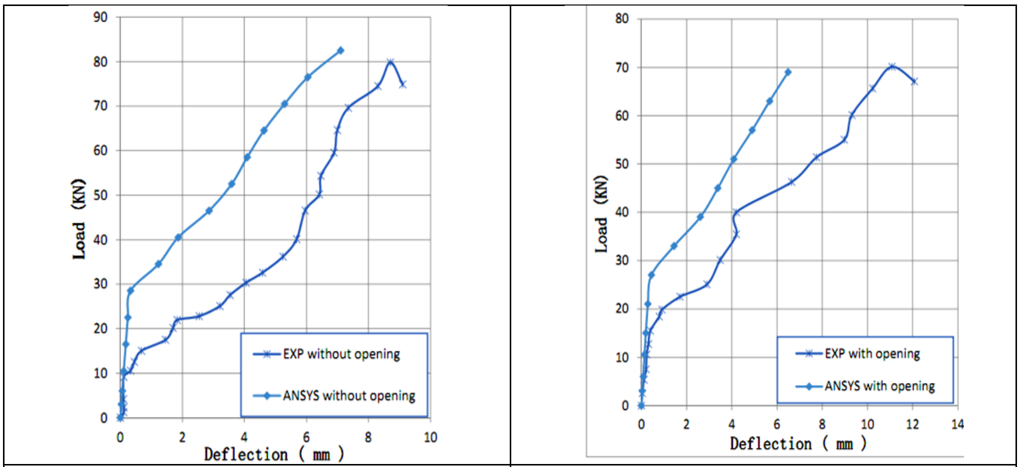


b) Concrete mesh model

Figure (11): The model of the control slab used in the nonlinear analysis.

4.3.2 Comparing between experimental and theoretical results

By using the experimental tested specimens with and without opening to find the best fitting of the theoretical curves as shown in figure 12. In general, by using nonlinear finite element program, the theoretical load-deflection curves shown that the slabs stiffness increase after cracking and their deflection decreases corresponding to the maximum loads.

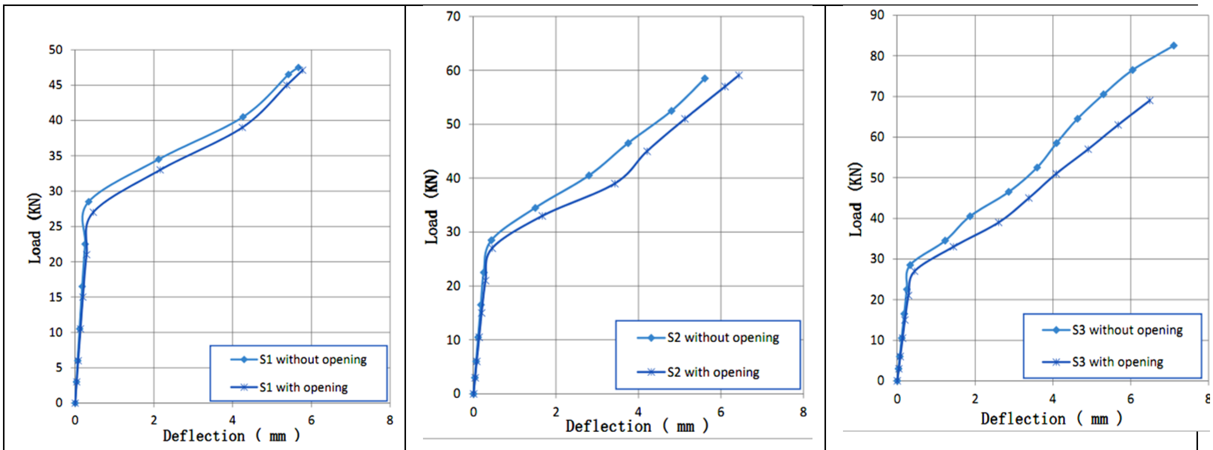


(a) Control slab without opening.	(b) Control slab with opening.

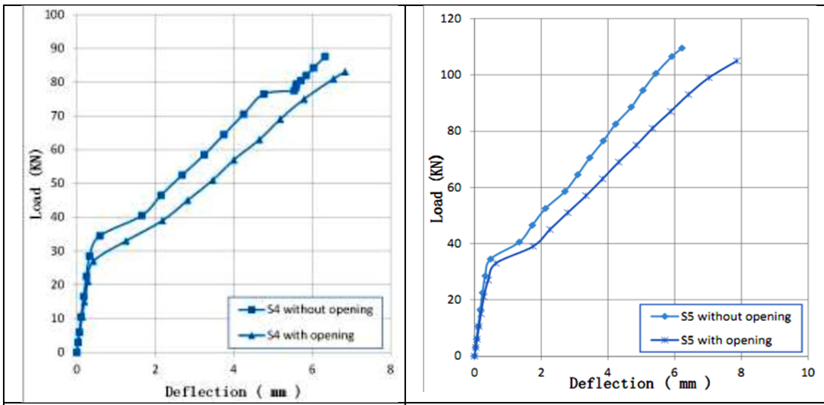
Figure (12): Theoretical and experimental load-deflection curves of control slab.

4.3.2 The effect of steel reinforcement ratio with opening in RC two way solid slab.

From the load deflection curves in figure 13 and table 5, for steel ratio (0.08% and 0.18%) which is less than the minimum steel ratio recommended by Egyptian Code, it was found that the opening did not have an effect on the load deflection curves. On the other hand, by increasing the steel ratio to (0.31%, 0.49% and 0.71%), there was a significant effect on the stiffness of slab especially after yielding it decreased. And also, by increasing the steel ratio, the rate of the decrease in the maximum load increased.



(a) RFT ratio 0.08 %.	**(b)** RFT ratio 0.18 %.	**(c)** RFT ratio 0.31 %.



(d) RFT ratio 0.49 %.	**(e)** RFT ratio 0.71 %.	

Figure 13: The effect of different opening at different steel ratios on the load deflection curves.

By comparing between the maximum load from yield line theory and nonlinear finite element program it was found that by increasing the area of steel, the difference between two values decreased. It was also noticed that the maximum load calculated by yield line theory is less than that by nonlinear finite element program.

Table (5): Comparing between theoretical analyses results which were done using ANSYS program and yield line theory.

Group	Steel ratio	P_{max} (yield line)		ANSYS results		
		Pattern 1	Pattern 2	P_{max} (kN)	$\Delta_{at\ max}$ (mm)	$A_{under\ curve}$ kN.mm
Slab without opening	0.08%	18.39	23.61	47.5	5.67	204
	0.18%	38.16	48.99	58.5	5.61	228
	0.31%	65	85.6	82.5	7.12	382
	0.49%	98.04	125.86	87.63	6.33	360
	0.71%	131.54	168.86	109.5	6.22	410
Slab with opening	0.08%	14.59	22.13	47.1	5.78	199
	0.18%	30.27	45.91	59.1	6.43	256
	0.31%	52.2	80.30	69	6.49	289
	0.49%	77.75	117.9	83.1	6.84	352
	0.71%	104.32	158.27	105	7.87	505

Figure 14 show the relation between the steel ratio and the area under p-δ curves. For slab without opening, it was found that the area under curves was affected by the minimum steel ratio recommended by different codes. On the other side, in the presence of the opening the rate of increasing of the area is constant and the effect of minimum steel reinforcement on the dissipation energy decreased.

The relations between load and deflection for RC slab different steel ratio were shown in figure 15-a. from figure finds that by increasing the steel reinforcement ratio, dissipation energy increased. The relations between load and deflection for RC slab with central square opening at different steel ratio were shown in figure 15-b. from this figure it was noticed that by increasing the steel reinforcement ratio, the deflection at the maximum load decreased and dissipation energy increased.

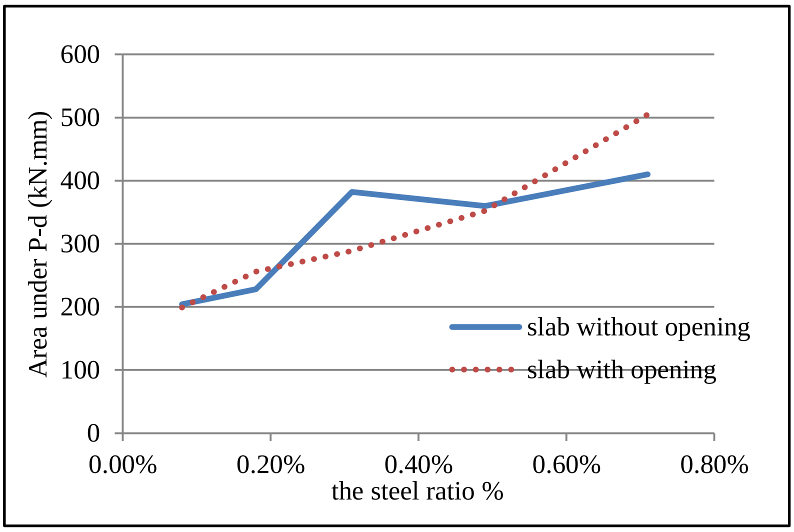
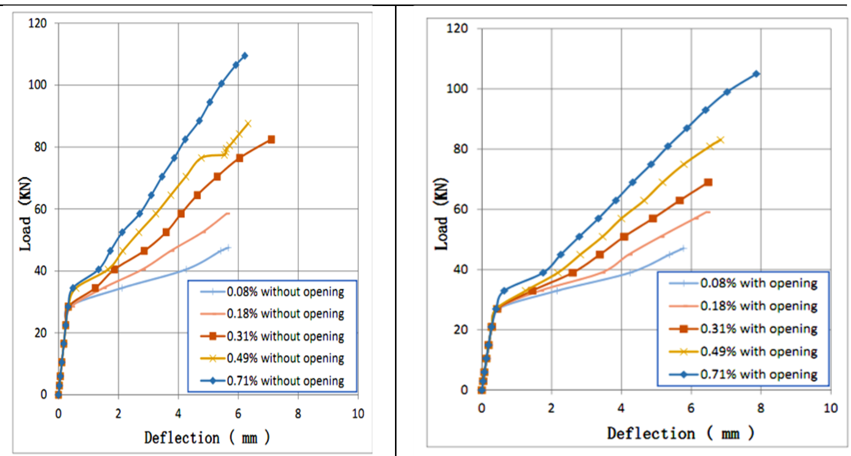


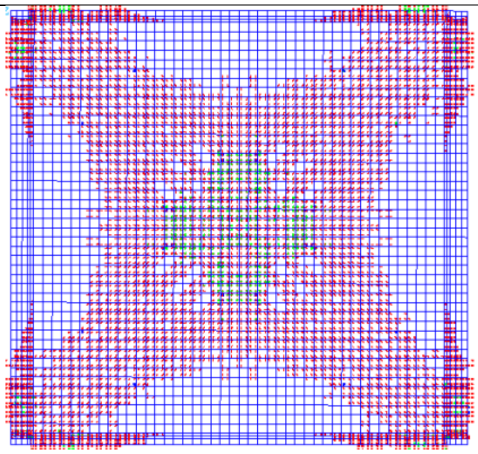
Figure 14: The relation between steel ratio and energy dissipation.



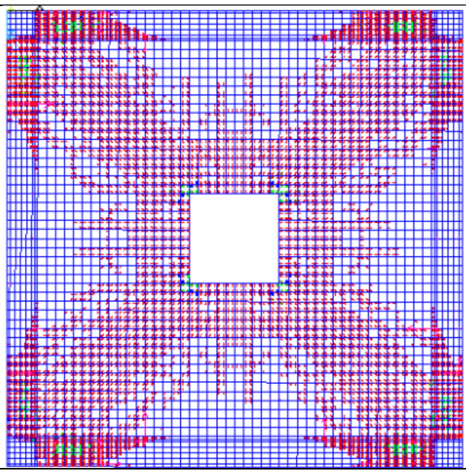
(a) Slabs without openings.	(b) Slabs with openings.

Figure (15): The effect of RFT ratio with and without opening on the load-deflection curves.

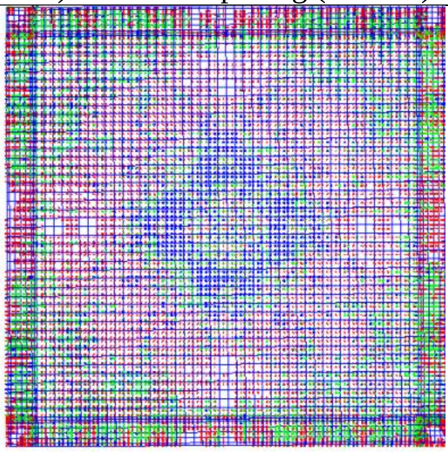
By comparing between the bottom crack patterns for reinforced concrete slab with and without opening it was found that the area of cracks decreased with opening (see figure 16) and the concrete crashing toward from under concentrated load to in slab with opening to corners for slab without opening.



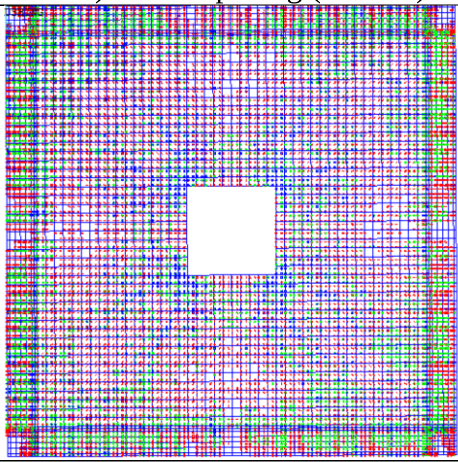
a) Without opening (34.5kN).



b) With opening (33.0kN).



c) Without opening at maximum load.



d) With opening at maximum load.

Figure (16): the effect of opening on the crack patterns.

5. THE CONCLUSION

1. The behavior of two-way solid slabs is affected by the presence of openings especially those in the mid spans of the slab the ratio between P_u of S2 to that of S1 was 0.878 which means that the opening caused a reduction in the slab load carrying capacity by 13%; meanwhile slabs are also affected by the method of openings introduction, especially if they were introduced after casting the ratio between P_{max} of S3 to that of S1 was 0.934.
2. For group one slabs the load carrying capacity was 79.8kN, 70.11kN and 74.57kN which was significant compared to the other groups as these slabs group was not strengthened; especially slab S3 as it was opened after casting the steel strap that was used to connect the steel bars has a good effect on transferring loads and maintaining slab strength. It is obvious that welding a steel strap to S3 cut bars at the edge of the opening affected the load carrying capacity as it was higher by 5.6% compared to S2.
3. In two way solid slabs, the energy dissipation significantly decreased when the steel ratio lessened than minimum (by 0.20%) causing a decrease in the strength.
4. The openings have significant effect in decreasing slab stiffness as the moment of inertia of the slab increase with its presence.
5. To avoid sharp decreasing in ductility, it is not recommended to decrease the minimum steel ratio than the code limitation.

6. Yield line analysis is an efficient method to calculate slab load carrying capacity as long as different assumptions of crack patterns are taken into consideration.

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