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Size Effect on Shear Strength of Concrete Deep Beams

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ملخص البحث

يحتوي هذا البحث علي دراسة نظرية بهدف الوصول لفهم افضل لظاهرة "نقص المقاومة مع زيادة الحجوم" المؤثرة علي مقاومة القص للكمرات العميقة. تقدم الدراسة النظرية النتائج التي تم الحصول عليها من تحليل تسع كمرات عميقة. حيث كانت المتغيرات الرئيسية في هذه الدراسة هي ابعاد الواح التحميل و الارتكاز, و عمق الكمرات , و النسبة بين عمق الكمرات و طول الواح التحميل و الارتكاز. سوف يتم عرض نماذج التحليل لهذه الكمرات والمقارنة بين نهج التصميم المختلفة الحالية و مدي تأثير هذه المتغيرات علي مقاومة القص و مناقشتها في هذه الدراسة.

Abstract

This paper presents a theoretical study to better understand the size effect phenomena on shear strength of concrete deep beams. The theoretical study presents the results obtained from the analysis of nine normal strength concrete deep. The main variables in this study were the loading and supporting area, beam's depth, and the ratio between beam's depth and plate length. The analysis models of the deep beams, and the comparison between different design approaches are presented in this paper. The effect of the studied variables are presented and discussed.

Keywords

Deep beam; Shear strength; Loading and supporting area; Size effect; Shear failure; Strut and tie.

1. Introduction

Deep beams are structural elements that are similar to beams in many ways but have smaller shear to depth ratios. Deep beams can have multiple useful applications in the construction of tall buildings, foundations, offshore structures, and several others.

The main force transferring mechanism of deep beams is a tied arch action, which can be described by the generation of a compression force in the web that in turn yields to a tension force in the perpendicular direction. Resulting from the tension forces diagonal cracks begin to form and propagate through the web, which decrease the load carrying capacity of the deep beam that is why shear capacity rather than flexure capacity usually dominates the strength of deep beams, providing a regular amount of longitudinal reinforcement is used.

Strut and tie is a very powerful design approach, and considered by many codes to be the only approach to design a reinforced concrete deep beams such as the ACI 318 and the EC2, although the ECP 203 has provided an alternative method (Empirical method) for deep beam design, it also considered the strut and tie to be the more general approach.

The size effect phenomenon was first introduced by Kani in the mid-1960s in his paper he demonstrated that safety factor for large beams could be 40 percent lower than for otherwise geometrically similar smaller beams.

Recently, several studies addressed the size effect phenomena. These analyses confirmed the theoretical prediction that there is indeed a significant size effect, but did not confirm the precise form of the size-effect law, so the need for further tests satisfying geometrical similarity conditions became apparent.

From what is mentioned above, a different approach is required to design deep beams.

2. A brief review of STM

In STM, reinforced concrete element is idealized as an equivalent truss, and analyzed for applied loads. The compression zones are represented as struts, while tension zones are converted into ties, which are in turn connected at the nodes to form a truss. With the main assumption that the ties will yield before the strut fails. The various components in a STM are struts, ties, and nodes. Struts are compression members and the different types of struts are shown in Figure 1, ties are the tension members and the represent reinforcing steel. Nodes form at points where struts and/or ties intersect. Nodes are described by the type of the members that intersect at the nodes. For example, a CCT node is one, which is bounded by two struts (C) and one tie (T). Using this nomenclature nodes are classified as CCC, CCT, CTT or TTT.

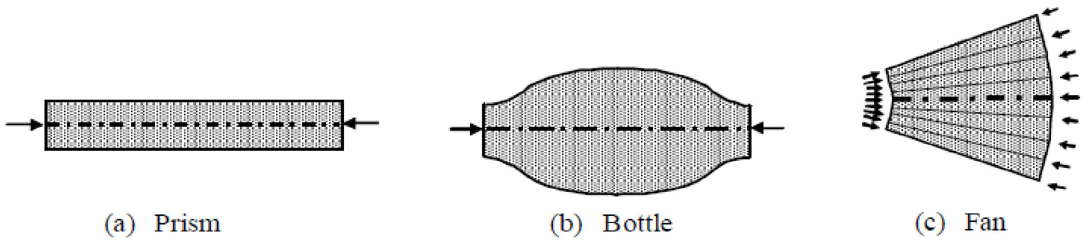

Figure 1, Strut shapes

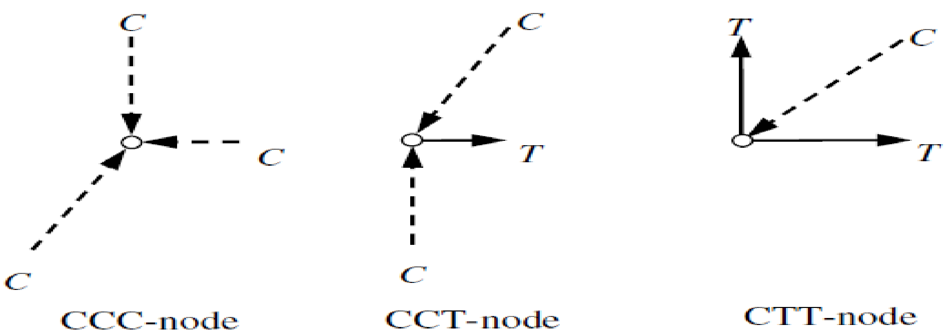

Figure 2, Different types of nodes

3. Analytical study and models

The codes that will be studied and used for comparison in this research are, Egyptian Code of Practice (ECP 203-2007), American Concrete Institute Building Code Requirements for Structural Concrete and Commentary (ACI 318-11), and Eurocode 2: Design of concrete structures (BS EN 1992-1-1:2004). Accordingly, CAST program will be used to as a tool to assist the comparison process between these codes and the experimental results.

3.1. Properties of studied beams

Three groups of concrete deep beams are designed, each consisting of three deep beams with overall height (h) varied from 400 mm to 900 mm, and loading and support plates lengths (w_p) from 60 mm to 180 mm. The beams will be studied under the effect of a two point load.

The clear span of the deep beams varied from 1200 mm to 2700 mm to keep the span to height ratio constant $(L/h = 3)$. The shear to span ratio was kept constant for all tested beams $(a/h = 1)$. All the studied beams were provided with flexure reinforcement ratio of approximately 2.0 %.

Deep beams dimensions and details are shown in Table 1, Figure 3 and Figure 4.

Table 1, Deep beams dimensions and details

Beam Name	Beam Width b (mm)	Beam Height h (mm)	Width of plates Wp (mm)	Span L (mm)	Shear Span a (mm)	Fcu (N/mm2)
B1-400/60	100	400	60	1200	400	30
B1-600/60	100	600	60	1800	600	30
B1-900/60	100	900	60	2700	900	30
B2-400/60	100	400	60	1200	400	30
B2-600/90	100	600	90	1800	600	30
B2-900/130	100	900	130	2700	900	30
B3-400/80	100	400	80	1200	400	30
B3-600/120	100	600	120	1800	600	30
B3-900/180	100	900	180	2700	900	30

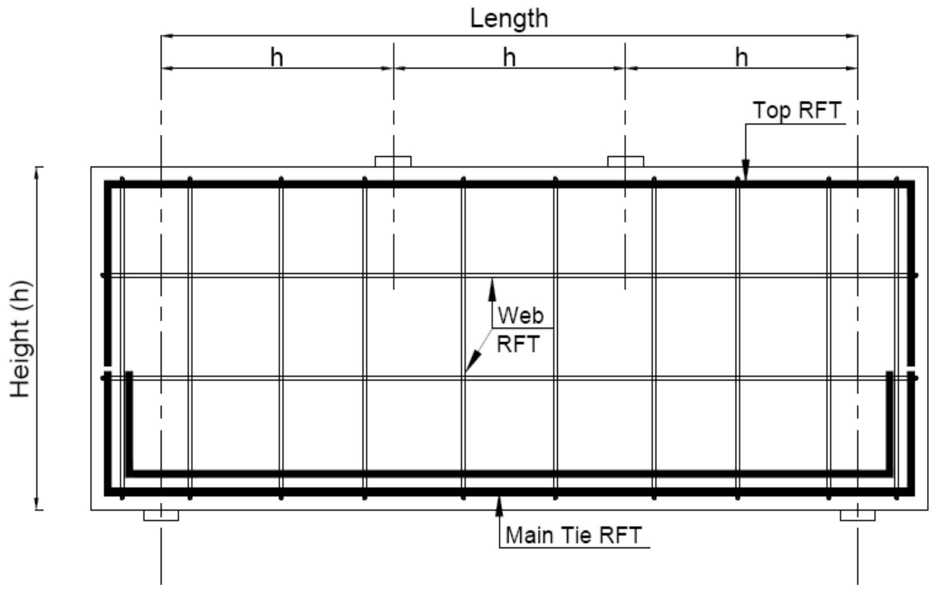

Figure 3, Deep Beams Typical Elevation

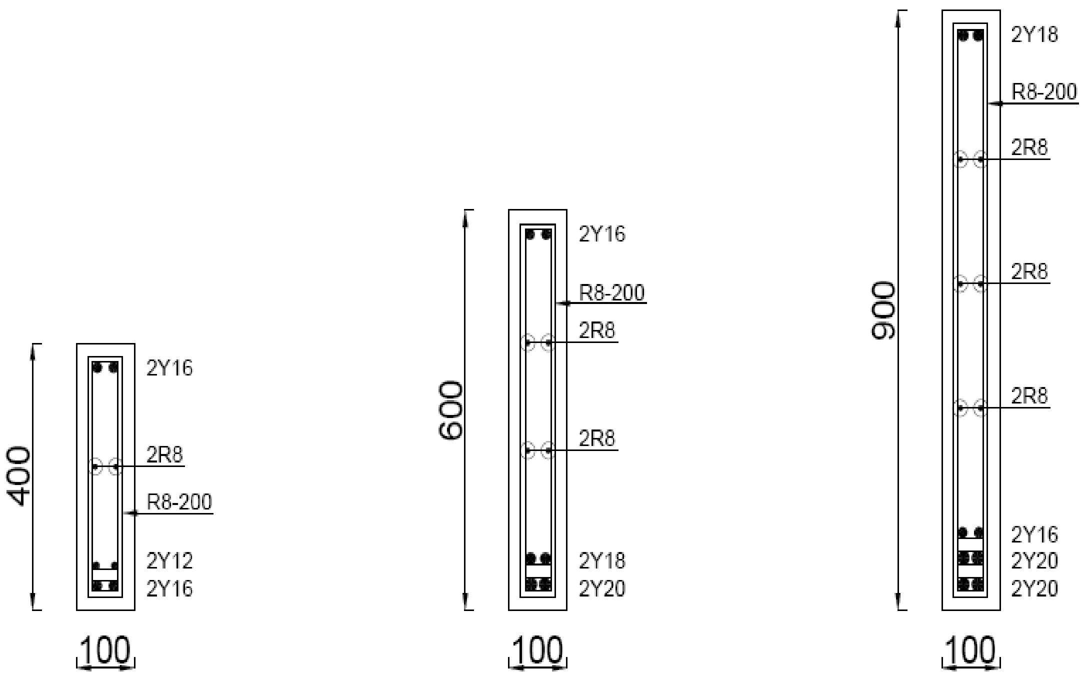

Figure 4, Deep Beams Sections and RFT Detail

Also as a part of this research a comparison between the analytical study and the experimental results of deep beams tested by Ning and Tan will be conducted.

Deep beams dimensions and details studied by Ning and Tan are shown in Table 2.

Table 2, Deep beams dimensions and details studied by Ning and Tan

Author	Beam Name	Beam Data							
		F_{cu} N/mm2	Width mm	Height mm	Shear Span mm	Plate Length mm	ρ_s %	ρ_{hz} %	ρ_{vl} %
Beams Studied By Ning and Kang (2007)	1DB35bw	32.4	80	350	350	52.5	1.25	0.40	0.40
	1DB50bw	34.3	115	500	500	75	1.28	0.39	0.39
	1DB70bw	35.4	160	700	700	105	1.25	0.45	0.45
	1DB100bw	35.9	230	1000	1000	150	1.22	0.41	0.41
	2DB35	34.8	80	350	350	52.5	1.25	0.00	0.00
	2DB50	40.5	80	500	500	75	1.28	0.00	0.00
	2DB70	31.0	80	700	700	105	1.25	0.00	0.00
	2DB100	38.3	80	1000	1000	150	1.22	0.00	0.00
	3DB35b	34.3	80	350	350	52.5	1.25	0.00	0.00
	3DB50b	35.4	115	500	500	75	1.28	0.00	0.00
	3DB70b	35.9	160	700	700	105	1.25	0.00	0.00
	3DB100b	36.6	230	1000	1000	150	1.22	0.00	0.00

3.2. Strut and Tie Model

All the three codes (ECP-203, ACI 318, and EC2) agree on the same criteria of what shall be designed by the STM and not follow Bernoulli hypothesis of plane strain distribution, they also agree on the STM components definition (struts, ties, and nodes).

Regardless that each code has its own strength formula and parameters, the difference can be found in determining the strength of the inclined struts, which have a great effect on the strength of the deep beam.

Table 3, shows the comparison between the codes according to the following criteria, deep beam definition, allowable stresses in strut-and-tie model, minimum web and main reinforcement, and code remarks.

Table 3, Strut and tie approach in different codes

Criteria	ECP 203-2007	ACI 318-11	Eurocode 2
Definition	$L/_d \leq 4$ *L : Beam span* *d : Beam depth*	$l_n/_h \leq 4$ *l_n : Clear span* *h : Beam total height*	$L/_d \leq 3$ *L : Beam span* *d : Beam depth*
Strength of Nodes	$F_{cn} = A_{cn} \cdot f_{cdn}$ *Where:* $f_{cdn} = 0.67\beta_n \frac{f_{cu}}{\gamma_c}$ $\gamma_c = 1.6$	$F_{nn} = \emptyset . f_{ce} . A_{nz}$ *Where:* $f_{ce} = 0.85 * \beta_n * f'_c$ $\emptyset = 0.75$	$\sigma_{Rd\,max} = k_n v' f'_c / \gamma_c$ *Where:* $v' = 1 - f'_c/_{250}$ $\gamma_c = 1.5$
C-C-C	$\beta_n = 1.0$	$\beta_n = 1.0$	$k_n = 1.0$
C-C-T	$\beta_n = 0.8$ *If the tie is mechanically anchored* $\beta_n = 1.0$	$\beta_n = 0.8$	$k_n = 0.85$
C-T-T or T-T-T	$\beta_n = 0.6$	$\beta_n = 0.6$	$k_n = 0.75$
Strength of Struts	$F_{cs} = A_{cs} \cdot f_{cds}$ *Where:* $f_{cds} = 0.67\beta_s \frac{f_{cu}}{\gamma_c}$ $\gamma_c = 1.6$ *For Reinforced strut* $F_{cs} = A_{cs} \cdot f_{cds} + A_s \cdot \frac{f_y}{\gamma_s}$ *Where:* $\gamma_s = 1.3$	$F_{ns} = \emptyset . f_{ce} . A_{cs}$ *Where:* $f_{ce} = 0.85 * \beta_s * f'_c$ $\emptyset = 0.75$ *For Reinforced strut* $F_{ns} = \emptyset(f_{ce}A_{cs} + A'_s f_y)$ *Where:* $\emptyset = 0.75$	*Prismatic strut* $\sigma_{Rd\,max} = f'_c/_{\gamma_c}$ *Where:* $\gamma_c = 1.5$ *Bottled strut* $\sigma_{Rd\,max} = 0.6v' f'_c/_{\gamma_c}$ *Where:* $\gamma_c = 1.5$
Prismatic strut	$\beta_s = 1.0$	$\beta_s = 1.0$	
Bottled strut	$\beta_s = 0.7$ or 0.6 *"see note 1"*	$\beta_s = 0.75$ or 0.6 *"see note 2"*	
Strut in tension	$\beta_s = 0.4$	$\beta_s = 0.4$	
Other cases	$\beta_s = 0.6$	$\beta_s = 0.6$	
Strength of Ties	$T_{ud} = A_s . f_y/_{\gamma_s}$ *Where:* $\gamma_s = 1.15$	$F_{nt} = \emptyset . A_{ts} . f_y$ *Where:* $\emptyset = 0.75$	$F_u = A_s . f_y/_{\gamma_s}$ *Where:* $\gamma_s = 1.15$
Minimum Main RFT	*No specific requirements for deep beams, regular beams regulations shall be applied*		
Minimum Web RFT	$\frac{A_v}{b.s_v} \geq 0.003$ *Mild steel* $\frac{A_v}{b.s_v} \geq 0.0025$ *High tensile steel* $\frac{A_h}{b.s_h} \geq 0.002$ *Mild steel* $\frac{A_h}{b.s_h} \geq 0.0015$ *High tensile steel* *Where:* s_v *or* $s_h \leq 200$	$A_v \geq 0.0025 b_w s$ *Where:* $s \leq 300\ mm$ $s \leq d/5$ $A_{vh} \geq 0.0025 b_w s_2$ *Where:* $s_2 \leq 300\ mm$ $s_2 \leq d/5$	$A_v \geq 0.1\%$ $A_{sv} \geq 150\ mm^2/m$ $A_h \geq 0.1\%$ $A_h \geq 150\ mm^2/m$ $s \leq 300\ mm$ *Note:* *The above mentioned values are in each face.*
Code Remarks	---	$V_u \leq \emptyset\sqrt{f'_c} b_w d$	---

- *Note 1: When the strut is parallel to crack direction use* $\beta_s = 0.7$*, if the strut is not parallel to cracks use* $\beta_s = 0.6$.
- *Note 2: For* f'_c *not greater than 40 MPa, and reinforcement crossing the strut satisfy Eq. (14), it shall be permitted to use* $\beta_s = 0.75$*, other wise use* $\beta_s = 0.6$.

3.3. Computer based STM software (CAST)

Another way to solve a strut-and-tie model is by using analytical software like CAST “Computer Aided for Strut-and-Tie”, it is a design tool that was developed by the University of Illinois since 2001, CAST program comprises with all strut-and-tie model elements.

The programme determines the type and strength of struts according to the ACI 318 as default option, although the programme allows for user defined strengths as well. When using CAST, in some cases a stabilizer is needed to equilibrate the model and make it numerically stable, a stabilizer is a strut with zero force, as shown in Figure 5 elements E9 and E10 are an example of stabilizers.

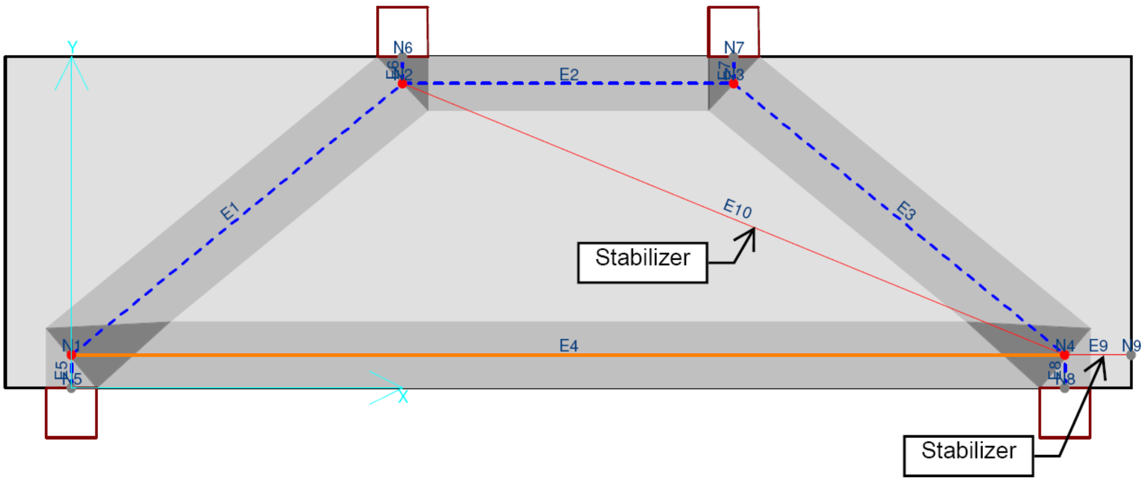

Figure 5, CAST program sample model

Figure 6, Figure 7, and Figure 8, are showing some the models made for the deep beams studied in this research.

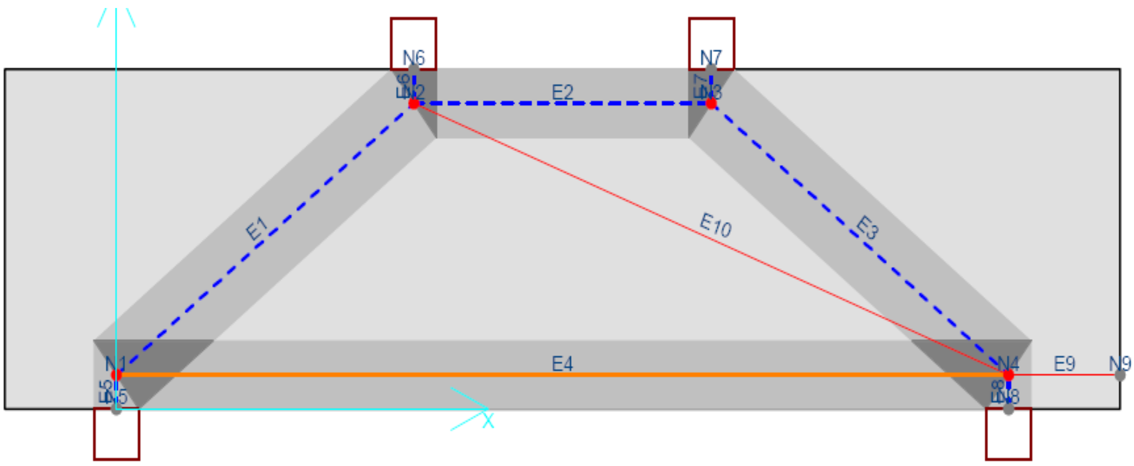

Figure 6, B1(400/60)

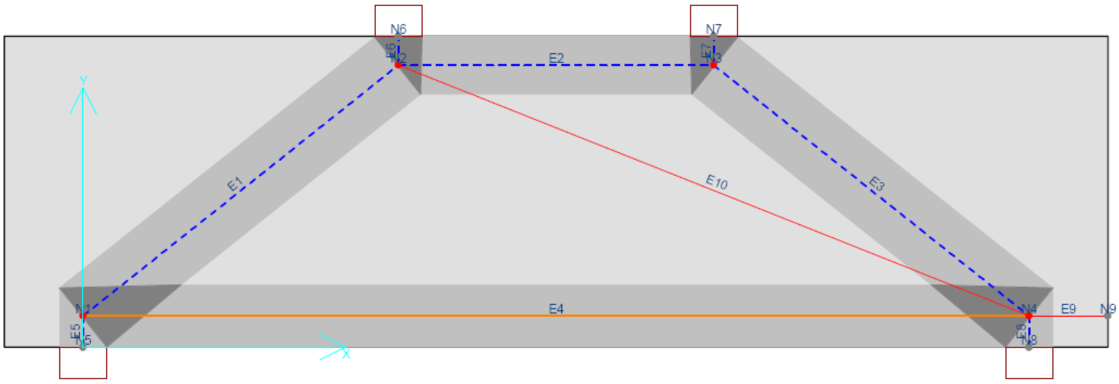

Figure 7, B2(600/90)

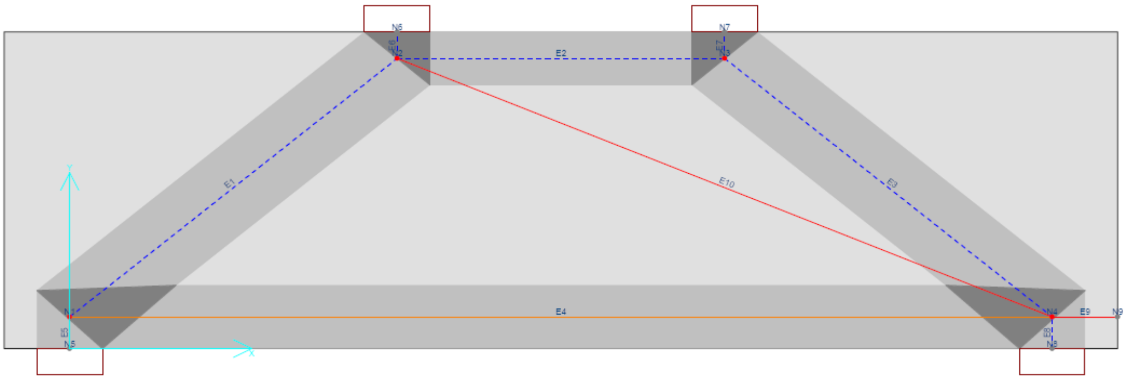

Figure 8, B3(900/180)

3.4. Size effect calculation

Based on the Mohr-Columb failure criterion, Tan et al proposed an equation for the ultimate shear strength (V_n), later this formula was improved for greater prediction consistency and accuracy, this formula is referred to as the "original STM". The proposed formula is:

$$V_n = \frac{1}{\frac{2\sin 2\theta_s}{f_t A_c} + \frac{\sin\theta_s}{f_c' A_{str}}} \qquad \text{Eq. (1)}$$

In Which:

Main Steel Web Steel Cracked Concrete

$$f_t = \frac{4A_s f_y \sin\theta_s}{A_c/\sin\theta_s} + \sum \frac{f_{yw} A_{sw}\sin(\theta_s+\theta_w)}{A_c/\sin\theta_s} + 0.31\sqrt{f_c'}\left(\frac{\varepsilon_{cr}}{\varepsilon_1}\right)^{0.4} \qquad \text{Eq. (2)}$$

$$\varepsilon_1 = \varepsilon_s + (\varepsilon_s + \varepsilon_2)\cot^2\theta_s \qquad \text{Eq. (3)}$$

Where:

$\theta_s =$ The inclined angle of diagonal strut.
$A_c =$ The cross sectional area of the deep beam.

$A_{str} =$ The cross sectional area of diagonal strut.
$f_t =$ The maximum tensile capacity of the bottom nodal zone.
A_s & A_{sw}= The respective total areas of longitudinal and web reinforcement.
f_y & f_{yw} =The respective yield strengths of longitudinal and web reinforcement.
$\theta_w =$ The inclined angle of web reinforcement with respect to horizontal line.
$\varepsilon_{cr} =$ The concrete strain at crack, $\varepsilon_{cr} = 0.00008$
$\varepsilon_1 =$ The principle tensile strain of concrete strut.
ε_2 & $\varepsilon_s =$ The respective tensile strain of longitudinal steel and peak compressive strain of concrete strut at crushing, $\varepsilon_2 = 0.002$

It was argued by many researchers that the traditional definition of ultimate shear strength of (V/bd) as indicative of size effect is unsuitable for concrete deep beams. As the origin of this equation comes from steel beams, with more or less uniform shear flow in the steel web, but, it does not reflect the arch action in a deep beam. After the formation of cracks, the shear strength of deep beams depends on the arch capacity. Where it self depends on the geometry and the boundary conditions of the strut. The authors define these variations as the secondary causes of size effect. To take account of these causes, the concrete strength at nodal zone was modified to vf_c', the term v is efficiency factor calculated as follows;

$$v = \xi . \zeta \qquad \text{Eq. (6.21)}$$

In which:

$$\xi = 0.8 + \frac{0.4}{\sqrt{1+(l-s)/50}} \qquad \text{Eq. (6.22)}$$

$$\zeta = 0.5 + \sqrt{\frac{kd_s}{l_s}} \leq 1.2 \qquad \text{Eq. (6.23)}$$

$$k = \frac{1}{2}\sqrt{\frac{\pi f_y}{f_{ct}}} \qquad \text{Eq. (6.24)}$$

Where:

$\xi =$ Account for the effect of strut geometry.
$\zeta =$ Account for the effect of strut boundary conditions influenced by web reinforcement.
l & s = Strut length and width respectively.
$k =$ Material factor incorporating steel bar yield strength f_y and concrete tensile strength f_{ct}.
d_s = Diameter of web steel bar.
l_s = The maximum spacing of web steel intercepted by the inclined strut.

When no web reinforcement is provided the following shall be applied:

- d_s: Will be the minimum diameter of bottom longitudinal steel bars.
- k: Will be half the above value.

Later Tan and Zhang proposed the following equation to calculate the ultimate shear strength taking account of size effect, referred to as "modified STM".

$$V_n = \frac{1}{\frac{2\, sin2\theta_S}{f_t\, A_c} + \frac{sin\theta_S}{v f_c'\, A_{str}}} \qquad \text{Eq. (6.25)}$$

4. Analysis results and comparison

The ultimate shear strength of all specimens had been calculated using the strut-and-tie models documented in the ECP 203-2007, ACI 318-11, EC 2, CAST program. Table 4 shows the obtained results.

Table 4, Comparison between obtained results

Beam Designation	Failure Load (kN)					
	P_{ECP}	P_{ACI}	P_{EC2}	P_{CAST}	P_{STM}	P_{mSTM}
B1-400/60	84	96	83	100	151	156
B1-600/60	110	125	108	110	224	228
B1-900/60	150	171	147	140	325	326
B2-400/60	84	96	83	100	151	156
B2-600/900	125	142	123	140	235	238
B2-900/130	185	211	182	202	352	351
B3-400/80	94	107	93	115	158	162
B3-600/120	139	159	138	158	244	247
B3-900/180	209	239	207	230	368	366

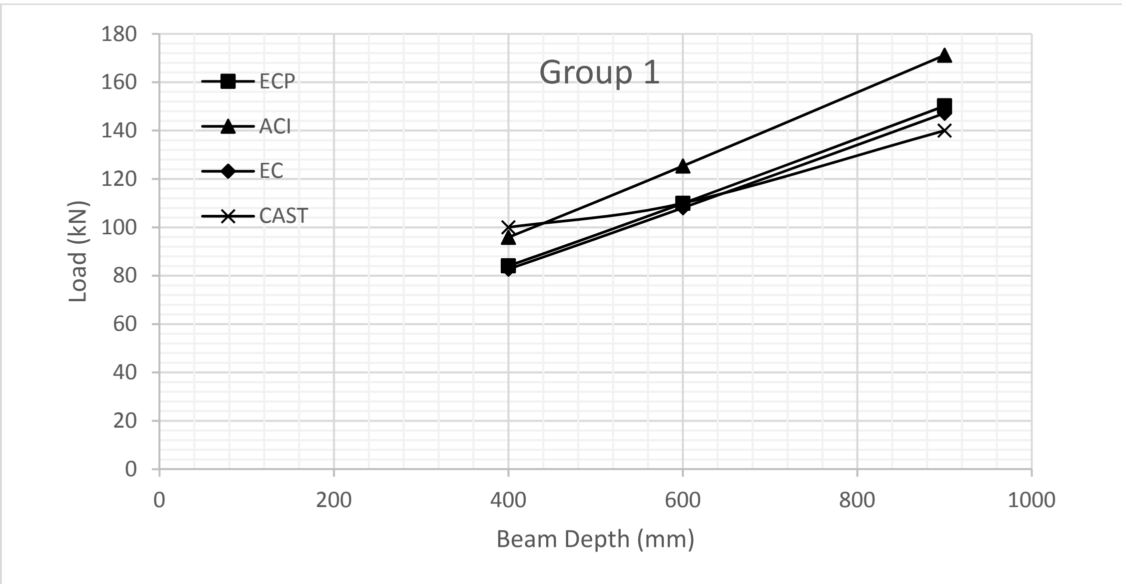

Figure 9, Group-1 Deep Beams

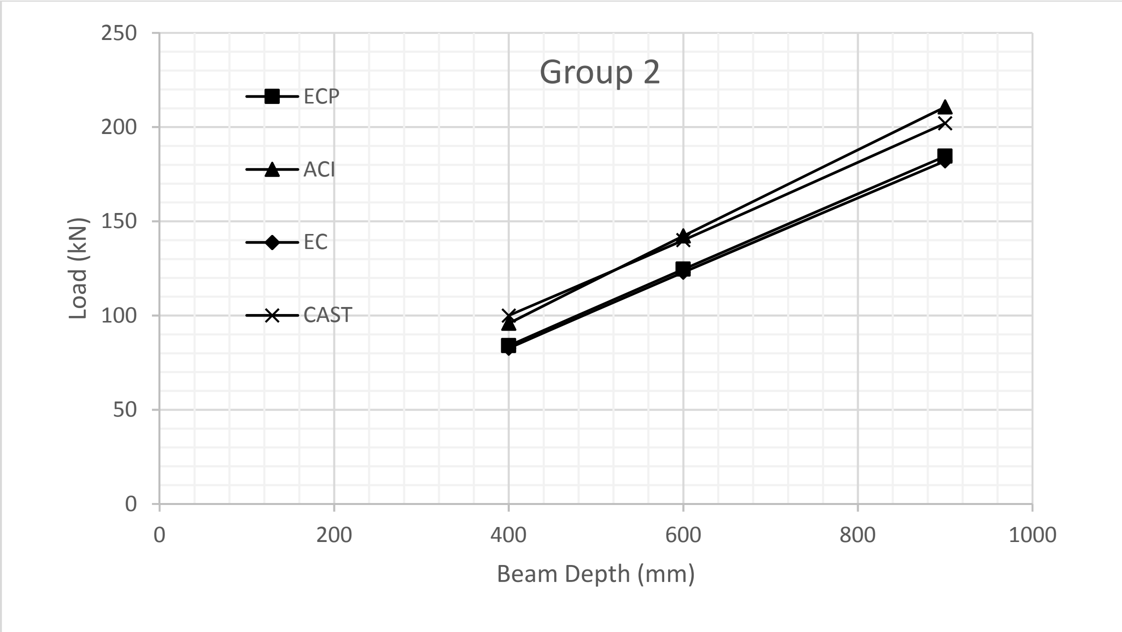

Figure 10, Group–2 Deep Beams

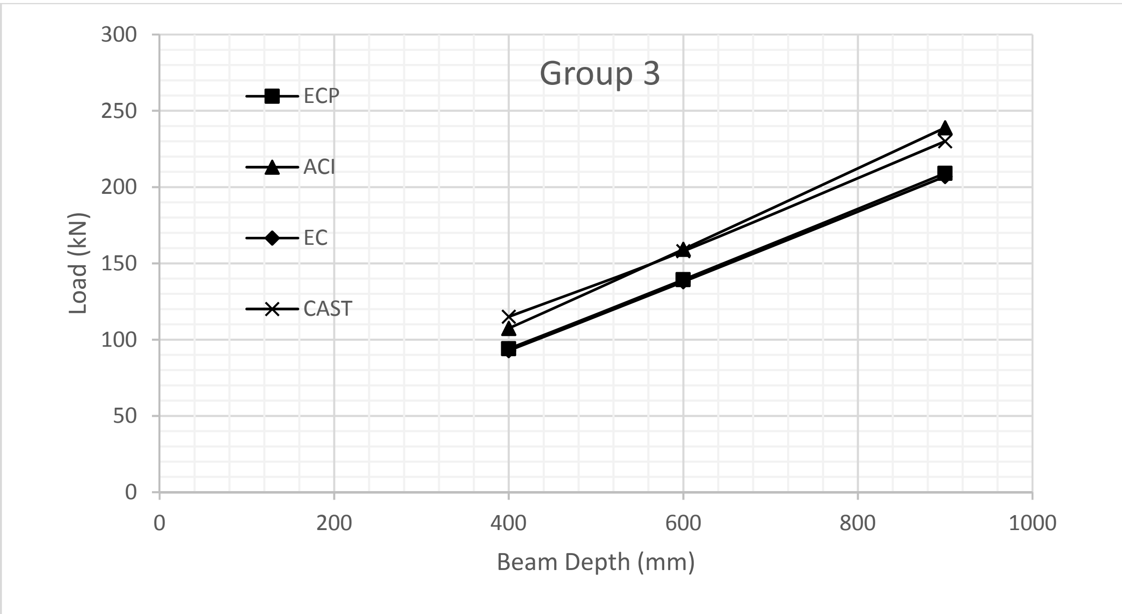

Figure 11, Group-3 Deep Beams

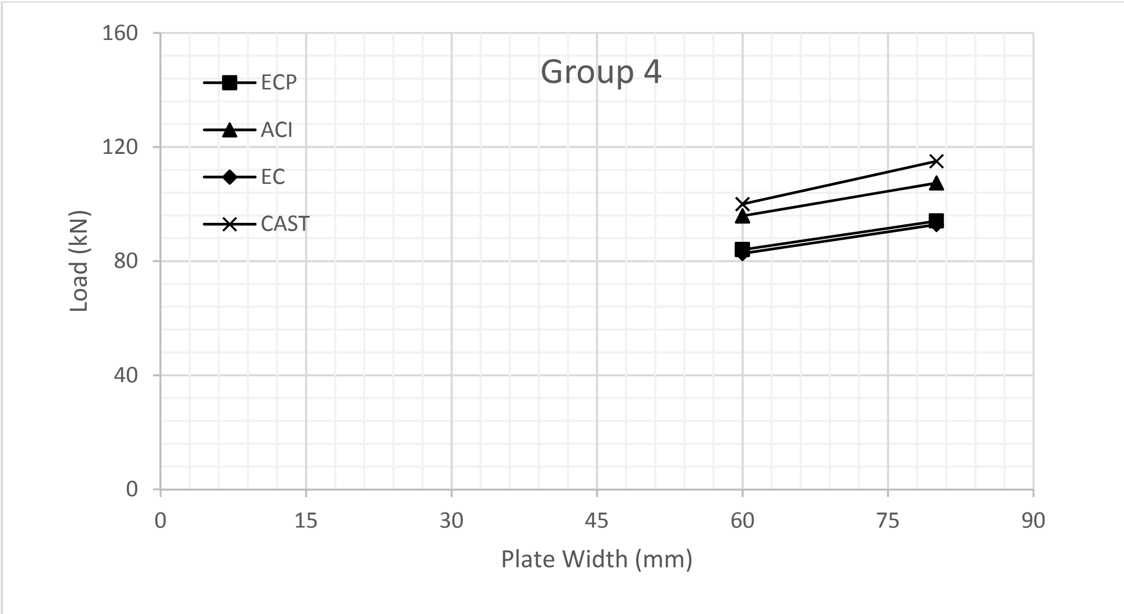

Figure 12, Group-4 Beam Depth 400mm

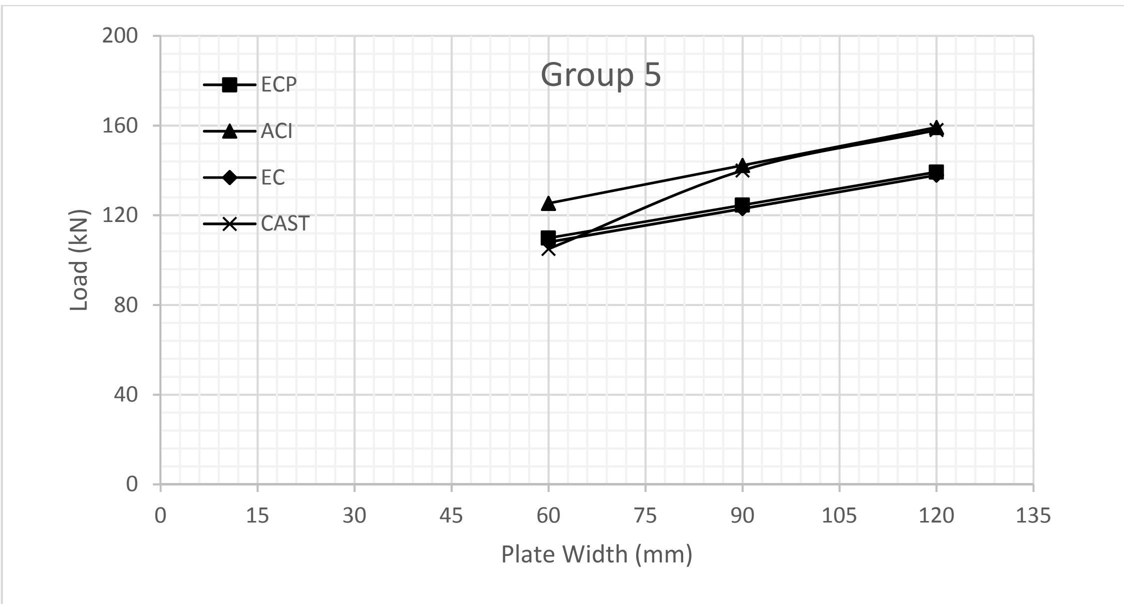

Figure 13, Group-5 Beam Depth 600mm

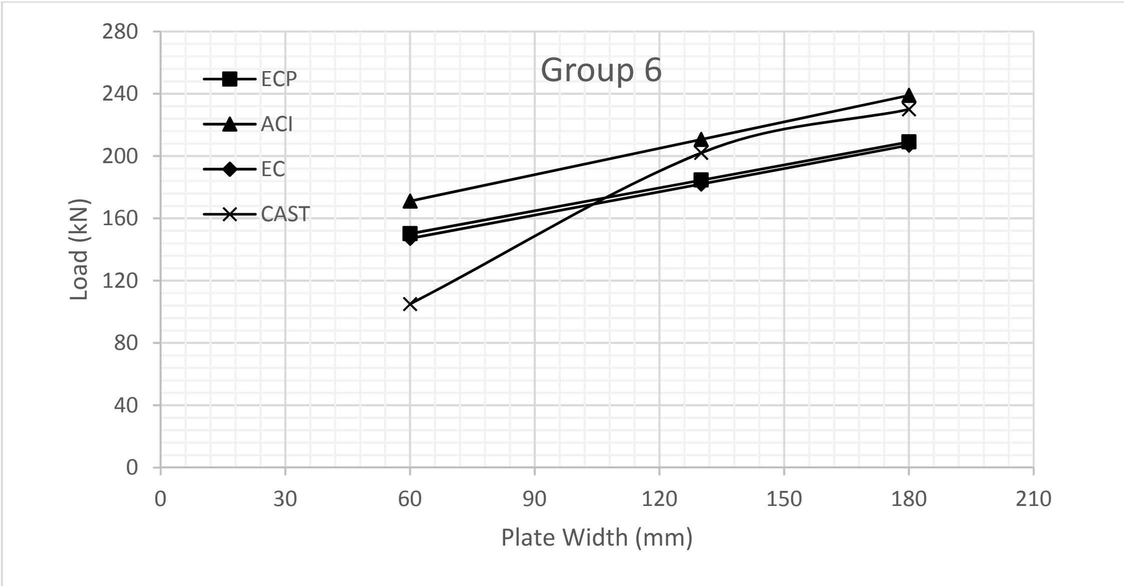

Figure 14, Group-6 Beam Depth 900mm

The analysis results and a comparison between analytical and experimental data for beams studied by Tan and Zhang are shown in Table 5.

Table 5, Deep Beams studied by Tan and Zhang analytical results

Author	Beam Designation	Failure Load (kN)						
		P_{Exp}	P_{ECP}	P_{ACI}	P_{EC2}	P_{CAST}	P_{STM}	P_{mSTM}
Beams Studied By Ning and Kang (2007)	1DB35bw	100	66	86	56	73	94	94
	1DB50bw	187	144	164	120	157	204	203
	1DB70bw	427	282	303	233	314	393	398
	1DB100bw	775	572	671	474	656	830	846
	2DB35	85	70	68	58	61	89	83
	2DB50	136	117	81	105	103	131	118
	2DB70	156	115	113	110	110	177	154
	2DB100	242	196	164	184	195	274	232
	3DB35b	85	69	69	58	61	89	83
	3DB50b	167	147	132	122	130	196	180
	3DB70b	361	283	243	234	254	376	344
	3DB100b	672	578	458	477	537	788	709

Author	Beam Designation	**ECP /Exp**	**ACI /Exp**	**EC2 /Exp**	**CAST /Exp**	**STM/Exp**	**mSTM/Exp**
Beams Studied By Ning and Kang (2007)	1DB35bw	0.67	0.86	0.57	0.73	0.94	0.94
	1DB50bw	0.77	0.88	0.65	0.84	1.09	1.09
	1DB70bw	0.66	0.71	0.55	0.74	0.92	0.93
	1DB100bw	0.74	0.87	0.61	0.85	1.07	1.09
	2DB35	0.82	0.80	0.68	0.72	1.04	0.98
	2DB50	0.86	0.60	0.78	0.76	0.97	0.87
	2DB70	0.74	0.72	0.70	0.71	1.14	0.99
	2DB100	0.81	0.68	0.76	0.81	1.13	0.96
	3DB35b	0.82	0.81	0.68	0.72	1.04	0.98
	3DB50b	0.88	0.79	0.73	0.78	1.17	1.07
	3DB70b	0.79	0.67	0.65	0.71	1.04	0.95
	3DB100b	0.86	0.68	0.71	0.80	1.17	1.06
Mean		0.78	0.76	0.67	0.76	1.06	0.99
Standard Deviation		0.07	0.09	0.07	0.05	0.09	0.07
COV		0.09	0.12	0.11	0.07	0.08	0.07

5. Conclusions and summary

1) Increasing deep beam depth does not mean increasing the shear strength as long as the loading and/or supporting area is unchanged.
2) The factors that influence the ultimate strength of a compression member govern the size effect, i.e. strut geometry and boundary condition.
3) ECP 203-2007 code STM equation showed a geed agreement with the EC2, and appear to be more conservative than those of the ACI 318-2011 code equations, and the CAST program.
4) With lowest COV of 0.07 the modified STM and CAST program have the most accurate shear strength predictions compared to other analysis approaches.
5) By properly configuring the dimensions of loading and support plates, size effect in ultimate shear strength can be significantly mitigated.

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